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NOVEMBER 1960

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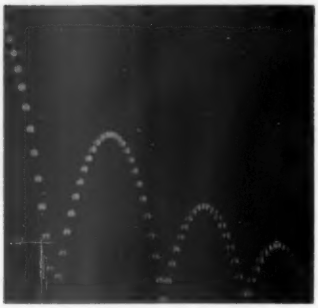

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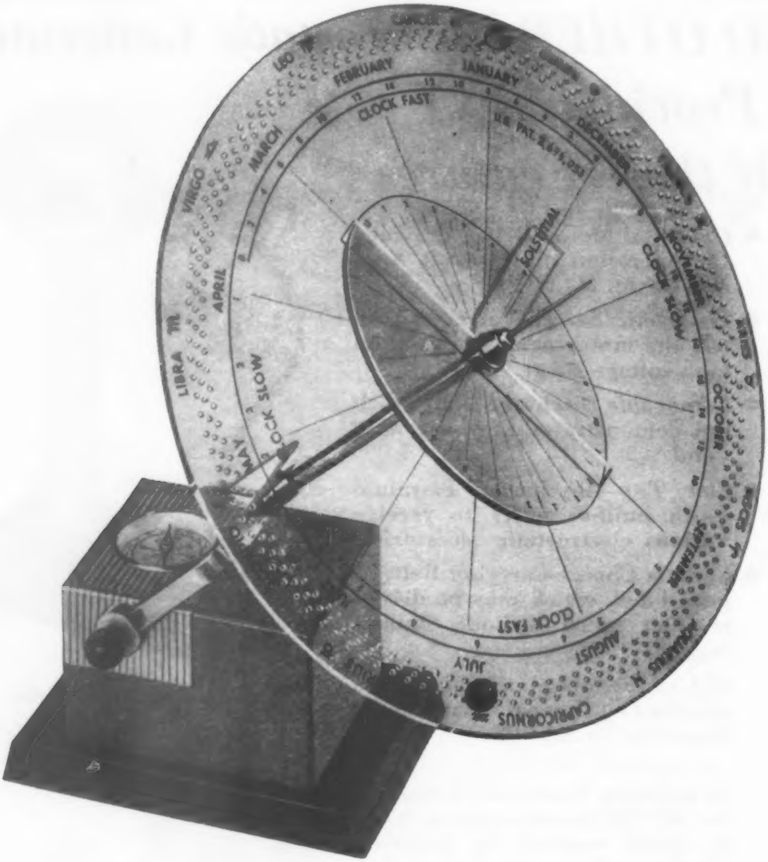

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Robert H. Goddard, Father of the Liquid Fuel Rocket

Enoch J. Haga
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Today there are probably few in America who would believe that it was an American who fathered the modern science of jet propulsion and rocketry; we are inclined to believe that the Germans began the race which we now run with the Soviet Union. But just forty-eight years ago an almost unknown American physics instructor at Princeton, fired with the great future he foresaw for the dreams that began to form in his brain, began a life of work that put the United States in the lead of nations engaged in rocketry. Had the full implications of this great man's work been realized—had he been taken more seriously—the United States might have been better enabled to maintain her first-place position.

As it was, this unknown instructor formulated a general theory of rocket action. Some of the essential features invented and patented by this man were used by the Germans in their V-2 rockets which rained down on Britain toward the end of World War II. Yet, when he had seriously proposed supersonic rockets of the V-2 type between the years 1914–1940, he had been ignored. As early as 1918 he had demonstrated the prototype of the now famous "Bazooka" rocket.

But in 1908, the year he received his B.S. from Worcester Polytechnic Institute, there was little to indicate that Robert H. Goddard (1882-1945) would emerge as America's first twentieth century astronaut. By 1910 Goddard received his A.M. from Clark University, and by 1911 his Ph.D. from the same institution. At the same time he

did his graduate work at Clark, he was physics instructor at Worcester Polytechnic. His doctoral dissertation had to do with coherers for receiving wireless messages. Even before receiving his B.S. he developed what is thought to be the first effective gyro-stabilizer for aircraft. He also worked with vacuum tubes and cathode ray beams. But in 1912 the imagination of this new Ph.D. began to take in the universe.

During the year 1912-1913 he was research instructor at Princeton University. His calculations had shown him that rockets would be ideal for reaching extremely high altitudes. Gun firing of projectiles, he demonstrated, would not produce the results of efficient rockets with tapered nozzles and motors strong enough to withstand high pressures. Efficiently utilized, only a little fuel would be required to lift payloads to great heights by rocket. Returning to Clark University in 1914 as physics instructor, Goddard began his experimentation. Starting with ship rockets, he went on manufacturing and experimenting with rockets of various kinds. In 1915 he became assistant professor at Clark. By 1916 he concluded that he could do no more with his own limited resources.

Appealing to the Smithsonian Institution for support, his eagerness and sincerity won him the attention of Dr. Charles D. Walcott, at that time Secretary of the Smithsonian. Goddard was commended for his report and asked how much money he would need. He thought he would need $10,000, actually asked for $5,000, and ultimately received $11,000. With this meager investment by today's standards, American space research was launched. During World War I, volunteering his services, Goddard was granted government aid under Smithsonian control for the purpose of developing long range military rockets. Research begun at Worcester Polytechnic Institute was continued in the shops of the Mt. Wilson Observatory in Pasadena, California. The fruit of the research carried on by Goddard and C. N. Hickman in these shops was demonstrated on the day before the Armistice in 1918 at the Aberdeen Proving Ground, Maryland. Various types of single-charge rockets, weighing from $1\frac{1}{2}$ to 17 lbs., propelled by double-base powder of 40% nitroglycerin and 60% nitrocellulose, were fired from hand-held lightweight launchers. Plans for 4″ rockets suitable for firing from aircraft were also revealed. Today it is easy to reflect upon the importance of that November demonstration some forty-two years ago, but next day, the war being over, rocket development ceased.

In 1919 Goddard's first published work in the field of rocket science was issued by the Smithsonian. Titled, *A Method of Reaching Extreme Altitudes*, it dealt with the potentialities of dry fuel or powder propelled rockets. In this same year Goddard was made full professor

at Clark. He was made Director of the Physical Laboratories at Clark in 1923.

At first Goddard had used black and smokeless powders for his rockets, but in 1920, deciding that liquid fuels would be more promising, he began using liquid hydrogen, oxygen, and hydrocarbons (gasoline). The first liquid fuel rocket ever fired left the earth on March 16, 1926; from this odd-looking instrument some ten feet high, all our liquid rockets of today—and the German V-2 of yesterday—are descended.

Colonel Charles A. Lindbergh became interested in Goddard's work in 1929, and he in turn was able to interest Daniel Guggenheim. Until his death Goddard was aided by Guggenheim or the Guggenheim Foundation, receiving annual grants in 1930–1932 and from 1934–1942. This help enabled Goddard to establish himself in a location favorable for experimentation, the Mescalero Ranch near Roswell, New Mexico. Great strides were made in this desert area, and in 1936 he summarized his achievements in another report issued by the Smithsonian, *Liquid-Propellant Rocket Development*. This report concluded with the words: "The next step in the development of the liquid-propellant rocket is the reduction of weight to a minimum. Some progress along this line has already been made."

World War II now intervened in Goddard's efforts. His old co-worker, C. N. Hickman, a Bell Telephone Laboratories engineer, was put in charge of the National Defense Research Committee in July of 1940. Hickman's first laboratory was at Indian Head, Maryland, and here Captain L. A. Skinner, U. S. Army Ordnance Department liaison officer with the National Defense Research Committee, spent much time; he had read reports of the 1918 Goddard and Hickman demonstrations at Aberdeen and became so interested in rocketry that he experimented with them himself from 1933 to 1940. It was under Skinner and Hickman that Captain E. G. Uhl developed the famous "Bazooka" at Indian Head. During the course of the war various institutions engaged in rocket research and development under contract with the National Defense Research Committee. Goddard himself again volunteered his services and did research in liquid rocket fuels for the Navy at Annapolis, Maryland. After the war Goddard planned to return to New Mexico where he hoped to establish some new and unheard of altitude records, but death cut short his brilliant career at the age of 62.

Goddard carried on much of his imaginative and ingenious work in the face of public indifference and lack of finances. But in spite of these difficulties, which were added to the technical obstacles which he faced, he saw his task through; almost by himself he created modern rocket science. The Board of Directors of the American

Rocket Society said of him: "The lifework of Dr. Goddard . . . will always remain a brilliant inspiration to all of those who are privileged to carry on his endeavors, and to every other bold explorer on the new frontiers of science. In time to come, his name will be set among the foremost of American technical pioneers."

Dr. Goddard himself once stated: "From my knowledge of the theoretical and experimental sides of the subject, I believe that a rocket from the earth will some day successfully reach one of the planets."

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THE CADI'S DECISION

On a warm August day, two wandering Persian beggars decided to stop beneath the branches of a tree and dine upon their combined gleanings. The taller one, Ali, drew out of his pouch three stale loaves and the smaller and more indolent, Zaid, took out two.

As they were about to begin their modest meal, a hungry stranger passed and asked to be permitted to join them. The two mendicants hesitated for a moment but quickly agreed when the stranger offered to pay for his food. After completing his repast, the stranger threw the beggar five gold coins and continued on his way.

No sooner did the man leave when a fierce quarrel broke out between Ali and Zaid. Ali who had given three loaves of the common table demanded three gold coins. While Zaid claimed with equal vehemence that inasmuch as they had agreed to share their bread, he deserved $2\frac{1}{2}$ gold coins.

As their argument grew hotter and they were about to resort to blows they were brought before the Cadi to decide the matter. The venerable judge listened carefully to the two opposing claims, stroked his beared thoughtfully and announced his decision: neither of the litigants was right. Ali, who had given three loaves, was to be awarded four gold coins while Zaid who had given two loaves was to get one gold coin. The spectators were amazed at the Cadi's decision, but when he explained further all agreed with the justice of his verdict. How did the Cadi explain his decision?

ANSWER

The Cadi reasoned that after the five loaves were divided each of the three particpants in the meal had $1\frac{2}{3}$ loaves to eat. As Ali had originally possessed three loaves, he had given up $1\frac{1}{3}$ loaves or four times as much as Zaid who had only given up a third of a loaf. Hence Ali was entitled to four coins and Zaid only one.

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Polyhedra of Any Dimension

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A polyhedron is defined, at least in our own three-dimensional world, as any solid bounded by planes. A four-dimensional polyhedron would, therefore, be any four dimensional figure bounded by solids; similar definitions applying to the polyhedra of any greater number of dimensions. This report will only deal with those figures which are regular or, at least, semi-regular.

The first case to be discussed will be that of the equilateral triangle and its expansion. The triangle in zero dimensions is simply a single point, and in one dimension it is simply a line. These two are present in the expansion of any figure, but here the similarity ends. In the second dimension, the triangle appears, and in the third dimension, the figure is a triangular pyramid or tetrahedron. At this point, it is necessary to bring in another series of important facts.

A line is formed by connecting a point with another point; a triangle, by connecting that line with another point properly placed; a pyramid, by connecting the triangle with still another point, again properly placed; and so forth.

As is seen, when the new point is connected with each of the old vertices, the number of new edges in the figure is equal to the number of these other vertices. So the number of edges in the new figure is equal to the number of edges in the previous figure added to the number of vertices in that figure. An expansion of this same property is applicable to all parts of the figure: vertices, edges, faces, etc.

This information may be summarized as follows, using V to represent vertices; E, edges; F, faces; and S, solids:

Parts:		V	E	F	S
Dimensions:	0	1			
	1	2	1		
	2	3	3	1	
	3	4	6	4	1

One notices that any number in this set is obtained by adding the number directly above it to the one immediately left of the latter number. This property even applies to the first column if a column of ones is assumed to be to the left of that column.

So far, we have discussed only the realm of three dimensions. We now proceed to realms of greater dimensions. The four-dimensional expansion of the triangle is formed, in continuation of the above dis-

* This is a paper written by Owen Bradford during his sophomore year for the Science Congress sponsored by the Suffolk County Section of the Science Teachers Association of New York.

cussion, by connecting a point to all vertices of a tetrahedron; a five dimensional expansion by joining a point to all vertices of the four dimensional object, and so forth. Perhaps it should be brought out here that no dimension higher than the third may really be obtained physically, but at least a representation of those dimensions may, in order to show general properties. All vertex angles in these figures must equl 60°, and obviously no more than three lines may be drawn at any given angle to each of the others.

So the set takes on its final form:

					Unnamed figures in:		
Parts:	*V*	*E*	*F*	*S*	*4-D*	*5-D*	
Dimensions: 0	1						
1	2	1					
2	3	3	1				
3	4	6	4	1			
4	5	10	10	5	1		
5	6	15	20	15	6	1	etc.

This new set has all the properties of the old; therefore, the expansion is complete.

A similar discussion applies to the expansion of a square. As was previously stated, the point and the line are present in all expansions. The second dimension of this particular expansion is the square, while the third dimension is the cube, or hexahedron. Again, a new group of properties must be brought in. The line is formed by connecting two points; a square by connecting corresponding vertices of two lines, appropriately placed; a cube, by connecting corresponding vertices of two squares, appropriately placed; and so on.

As the particular figure of any dimension is obtained by taking two figures of the previous dimension, the number of edges in the new figure is partially obtained by multiplying the number of edges in the original figure by two. Then, since corresponding vertices have been connected, there are as many additional edges as there were vertices in the original figure, as each vertex now has one and only one more edge radiating from it than it previously had. Again, an expansion of this property is applicable to all parts of the figure.

This information may be summarized as follows:

Parts:	*V*	*E*	*F*	*S*
Dimensions: 0	1			
1	2	1		
2	4	4	1	
3	8	12	6	1

Any number in this new set is obtained by *doubling* the number directly above and adding to that quantity the number to the left

of the upper one. Here, only a column of zeros must be assumed to the left of the first column to make it complete.

By connecting corresponding vertices of two cubes, one obtains a four dimensional cube, and by connecting those of two of these new figures, five dimensions result. Again, when I say the fifth dimension results, I mean a representation of that dimension.

Therefore, the set for the expansion of a square appears in its final form:

					Unnamed figures in:		
Parts:	*V*	*E*	*F*	*S*	4-*D*	5-*D*	
Dimensions: 0	1						
1	2	1					
2	4	4	1				
3	8	12	6	1			
4	16	32	24	8	1		
5	32	80	80	40	10	1	etc.

Up to now, all the figures discussed could apparently be expanded to any dimension. However, to my knowledge, these are the only two regular figures which may. Following is a discussion of the reason that, for example, a pentagon may not be expanded.

The set for a pentagon, through the second dimension, is:

Parts:	*V*	*E*	*F*
Dimensions: 0	1		
1	2	1	
2	5	5	1

By extending the formulas for any given member of the two previous sets, one sees that a row of negative ones must be assumed to the left of the first column. Also, it is seen that he multiplies the number above any member by 3, adds to it the number to the left of that, and resulting is the desired number. In this way, the row for the third dimension would read:

3	14	20	8	1

However, the only regular three-dimensional polyhedron with eight faces is the octahedron, and this does not qualify for two reasons. First, the other figures of the row do not apply; and second, the faces are not pentagons anyway. Obviously, one cannot just ignore the third dimension. It is interesting to note, however, that this set of numbers (14, 20, 8, 1) does satisfy the famous formula of Euler; that is:

$$V - E + F = 2$$

This formula is applicable to any polyhedron, regular or otherwise.

Of course, one need not confine himself to regular figures. The expansion of a rectangle or of a parallelogram has the same properties as a square has, so that expansion of these figures follows readily from that of a square.

However, difficulty does arise in a discussion of completely irregular three dimensional polyhedrons, and for this reason, I have not even attempted, to discuss them in this report.

RESISTANT BACTERIA RAISE HOSPITAL DEATHS

The most important cause of severe infection and death in hospitals today are antibiotic-resistant bacteria. These now top the previously more fatal pneumonia bacteria and strep-infection type bacteria that proved susceptible to antibiotic.

In Boston City Hospital in 1935, antibiotic resistant Staphylococcus aureus, which causes boils, carbuncles and other inflammations, accounted for one of every five cases of infection and less than one of every five cases of death from bacteria-caused infections. In 1957, it accounted for two-fifths of both infections and deaths.

The reliance on antibiotics may be responsible for increasing laxity in application of strict aseptic methods for avoiding infection and cross-infections in surgery, nurseries and general wards in which infections are being treated. This laxity is undoubtedly a major contributing factor, to the rise in staph incidence and fatality.

No remedies for stemming the continuing rise in staph deaths has yet been demonstrated to produce any lasting salutary effect.

ORBITING JET SEEN AS "ULTIMATE AIRPLANE"

The "ultimate airplane" which will fly 17,000 miles an hour or 25 times the speed of sound, was described by Alexander Kartveli, a Russian-born aircraft engineer responsible for the development of 14 aircraft types.

Mr. Kartveli, now vice president of research and development for Republic Aviation Corporation, stated that the craft would be the last major type before aircraft makers turn most of their attention from aviation to astronautics.

It will be 170 feet long, 33 feet high and have a wingspan of nearly 99 feet. Its front will be a massive series of air ducts to draw huge quantities of air into four hydrogen-burning engines in combination with four ramjets.

It will be capable of taking off from the ground with a substantial military load, accelerating to orbital speed, orbiting around the earth and landing on earth when desired.

Mr. Kartveli said design consultants have already done basic exploratory work on the combustion scheme to power such a craft.

He also presented designs for three generations of planes he believes will come before the "ultimate":

1. A fighter-bomber, capable of vertical take-off, that could fly at 1,500 miles an hour. It could pinpoint bombs on a target from 75,000 feet.
2. A nuclear-ramjet strategic bomber, to be produced between 1970 and 1975, that would fly at 2,800 miles an hour and cruise at 85,000 feet. Its two-man crew would be in a shielded compartment. This bomber would eliminate U. S. reliance on allied bases.
3. A 4,960-mile-an-hour bomber, shaped like a great triangle, capable of of flight at 120,000 feet. With two ramjet and two turbojet engines, it would have a range of 5,000 miles.

Using the Cathode-Ray Oscilloscope in the High School Trigonometry Classroom*

Matthew H. Bruce, Jr.

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A most advantageous combination of subjects available for an instructor in mathematics and science is found in that of physics and trigonometry, taught concurrently. In the conventional high school program, physics taught in the senior year and trigonometry taught the first semester of the senior year provide many advantages over other arrangements. For example, early in the sequence of high school physics the study of mechanics is a necessity. This cannot be done effectively, for college preparatory students at least, without having trigonometry available as a tool. Try some time to teach resolution of forces or simple harmonic motion without trigonometry.

The advantages of the teaching of physics and trigonometry concurrently by the same instructor, as opposed to the frequently employed system of physics in the junior year and trigonometry in the senior year are obvious. The two "subject" areas are irrevocably tied together, and it must be insured that the students do not fall into the "never the twain shall meet" attitude that so frequently isolates what goes on in one classroom from what goes on in another. Naturally, the arrangement described above cannot possibly be practical in every school situation, but the two areas can be firmly cemented together in the minds of the students.

The use of trigonometric methods in the physics classroom is, or should be, thoroughly practiced, but the relationship between physics and trigonometry is a two-way street and the traffic in one direction is often unfortunately light. One means of stimulating the flow in that direction is presented here.

In teaching the graphic representation of trigonometric functions and the graphing of more complex trigonometric expressions by the method of adding ordinates, the cathode-ray oscilloscope can be a teaching aid of great value. Its use provides a most graphic demonstration of trigonometric relationships representing natural phenomena, a concrete means of relating the study of trigonometry to its applications in the fields of electricity, electronics, and sound, which do not generally make their appearance in the high school trigonometry classroom. It can do much to lift trigonometry out of the "device for solving right triangles" rut.

Several areas for discussion seem suitable:

* This report was an entry in the 1960 STAR (Science Teacher Achievement Recognition) awards program conducted by the National Science Teachers Association and sponsored by the National Cancer Institute, U. S. Public Health Service.

(1) Demonstrating the periodicity of trigonometric relations as they occur in nature;
(2) Illustrating the ideas of amplitude, period, and phase in the graphic representation of trigonometric functions;
(3) Demonstrating the combination of trigonometric expressions by the method of adding ordinates, and applications of this concept such as the square and sawtooth wave forms, harmonics, beats, and heterodyning.

Other areas present possibilities, of course, and while this discussion will be limited to the three areas mentioned above, it is hoped that others will occur to the reader.

One word of caution (probably unnecessary): this type of demonstration has a high entertainment value, which should not be allowed to dilute the purpose of the work. High school age youngsters, even in this day of household electronics, are fascinated by apparatus of the type used, and some will sit happily watching, if allowed to do so, without a trigonometric thought entering their heads.

The Cathode Ray Oscilloscope

While the actual operation of the oscilloscope will not be discussed here except as directly concerned with the concepts under examination, some basic ideas of the oscilloscope's operation from a trigonometric point of view must be presented.

In its simplest employment, the oscilloscope is an electronic device for the visual, graphic display of periodic functions. An ordinate, representing the instantaneous voltage of a periodically repeating external signal, is plotted against an abscissa, or internal sweep signal, representing time. The source of the external signal may vary considerably in different usages, but as used here will consist of sinusoidal or compounds of sinusoidal wave forms provided by alternating current electricity or sound.

The internal sweep voltage, generated within the oscilloscope, builds up from a negative value, through zero, to a positive value, and by deflecting a stream of electrons shot at the face (or screen) of the tube from inside, causes the stream to trace a moving fluorescent spot from left to right across the screen, in a measurable length of time. The trace returns to its starting point virtually instantaneously, drawn by a near-instantaneous change in the sweep voltage from positive to negative. This process is repeated over and over. Persistence of the image on the screen causes the moving point to appear as a horizontal line across the screen, giving a continuous horizontal time axis analogous to the x-axis in the ordinary graphic representation of trigonometric functions.

The external signal deflects the stream of electrons in a vertical direction, causing the spot to trace a graph of the function representing the external signal on the screen. By adjusting the time needed for the horizontal trace to cross the screen, a single period or as many periods of the external signal as desired may be displayed. The amplitude of the graph of the function may be varied also, as may the spread of the trace across the screen.

For the person unschooled in the theory and use of the oscilloscope, many adequate references are available. Several of these are included in the bibliography.

Periodic Functions

Of the concepts mentioned previously, periodicity is perhaps the simplest to demonstrate and relate to the graphs of angle functions as commonly presented in high school trigonometry. It is assumed here that the simpler graphs of trigonometric functions, such as $y = \text{sine } x$, etc., and the ideas of period and amplitude have been already introduced.

Adjust the internal sweep frequency to sixty cycles per second. A low voltage sixty cycle per second alternating current across the vertical input, as shown in Figure 1a, will then produce a single period of a sinusoidal wave form. (Some oscilloscopes have a built-in sixty cycle test tap providing about six volts alternating current.) Minor adjustments of the sweep frequency fine control and the introduction of a small amount of internal synchronization signal will be necessary to "stop" the picture. Care should be taken to use as small an amount as possible of synchronization signal, since in many 'scopes the use of larger amounts of synchronization signal tends to distort the wave form. Actually, the wave form of alternating current is not a true sine wave form, but it is sinusoidal and thus serves the purpose here.

A similar effect may be obtained by using the microphone and amplifier, as shown in Figure 1b, and a tuning fork, to provide the trace of the wave form.

Since the horizontal trace moves across the tube face with a period that is made to exactly match that of the sixty cycle external signal, the display is that of a single period of a sine curve which coincides with the trace of the period that went before, and with that of the period that went before that, etc. The resulting steady picture indicates the exactness of the periodic nature of the alternating current.

The one-sixtieth-of-a-second period of the alternating current is analogous to one revolution of a radius vector, or two π radians, and the wave form of the trace corresponds to the curve generated by the successive ordinates of the radius vector through one revolution

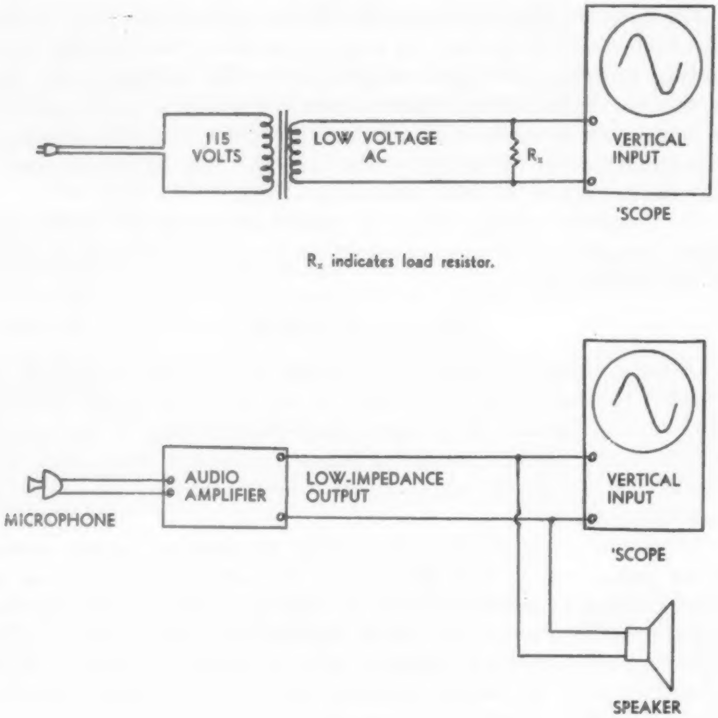


FIG. 1

starting at zero degrees in standard position. Thus, the trace illustrates the graph of $y = a$ sine x, where a represents the amplitude, proportional to the voltage applied across the vertical input.

AMPLITUDE, PERIOD AND PHASE

Using the apparatus set up as illustrated in Figure 1b, the ideas of amplitude and period can be demonstrated. The microphone, amplifier and oscilloscope, and speaker as shown will give both visual and audible presentation. If only the visual display is desired, the speaker and even the amplifier may be omitted, depending on the type of microphone and 'scope used. The external amplifier is usually desirable since it gives somewhat more flexibility of control.

With the sweep frequency set at about 256 cycles per second, strike a C' (256 cps) tuning fork lightly and hold it in front of the microphone. Adjust the frequency and synchronization controls to obtain a steady picture of a single period, and set the gain of the amplifier

to give a vertical height of about one-third the height of the tube face. (It may be necessary to strike the fork again to make the adjustments and display the resulting wave form.) Without altering the control settings, strike the tuning fork harder than before and note the increased vertical height of the trace, indicating increased amplitude of vibration of the source. Point out the analogy to the graphs of $y = \text{sine } x$ and $y = 2 \text{ sine } x$. (See Figure 2, a and b.)

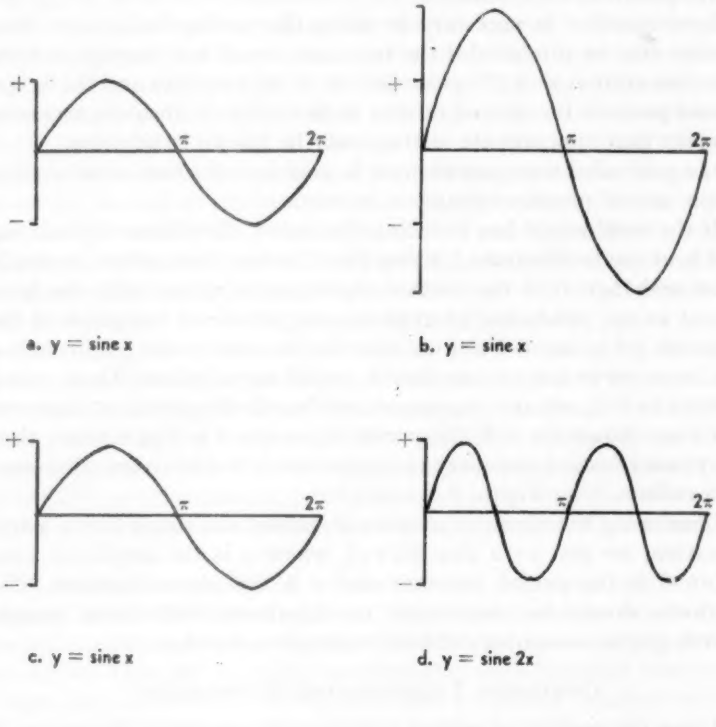


FIG. 2

Again without altering the frequency or gain settings, strike a C'' (512 cps) tuning fork and hold it in front of the microphone. Note the number of periods appearing over the same time length of the horizontal trace, compared to that of the C' fork. Using a fork of double the original frequency produces a period of half the original period. This corresponds to the graphs of $y=\text{sine } x$ and $y=\text{sine } 2x$. (See Figure 2, c and d.) In this manner, frequency and period are shown to be reciprocal to each other.

Some practice is necessary in using the tuning forks, since harmonics may be produced if the forks are struck too sharply. A little experimentation with the gain controls of the amplifier and the 'scope should produce the desired results. Rehearsal is an absolute necessity in order that approximate settings may be known in advance.

If a grid-ruled transparent mask is used over the face of the oscilloscope, actual measurements can be made.

If the oscilloscope has a phase adjustment, the effects of phase lag and lead can be illustrated. Using the C' tuning fork, obtain a steady trace and then shift the vertical signal out of phase with the horizontal sweep, producing an approximate picture of the graph of the equation $y=\text{cosine } x$. Point out that the sine and cosine graphs follow the same curve but are one-fourth period out of phase. Thus, cosine $x=\text{sine }(x+c)$, where c represents one-fourth the period in whatever units are chosen for x. If the period of $y=\text{sine } x$ is 2 pi radians, then the phase constant necessary to equate sine $(x+c)$ to cosine x becomes $\frac{1}{2}$ pi radians. (See Figure 2, e and f.)

Combining the effects of amplitude, period and phase into a single equation, we get: $y=a \text{ sine } b(x+c)$, where a is the amplitude constant, b is the period constant and c is the phase constant. The students should be encouraged to experiment with some sample sketch graphs, assigning different values to a, b and c.

Combining Trigonometric Expressions

Using the method of adding ordinates to compound the curves of the expressions $y=\text{sine } x$ and $y=\text{sine } 2x$, illustrate graphically the wave form of the tone produced by a C' (256 cps) tuning fork and a C'' (512 cps) tuning fork sounded simultaneously. After predicting the result graphically, actually perform the experiment with the apparatus as shown in Figure 1b, and compare the wave form displayed on the oscilloscope with that predicted. If the internal sweep frequency is matched to that of the C'' fork, two periods will be displayed. Some variation will occur because of differences in amplitude of vibration of the two sources, but practice in the timing of striking of the two forks should enable the demonstrator to obtain a very reasonable wave form match.

The use of resonance boxes, found in nearly any high school physics laboratory, may aid in obtaining sufficient amplitude for a good visual display even without the use of an external amplifier.

A further demonstration of this idea, using a small open-end organ pipe of the type usually found in the physics laboratory, will tie the idea in closely with the physical phenomena ordinarily studied in a typical high school physics course. Such a pipe, when blown sharply, will sound both its fundamental (first harmonic) and its second harmonic at the same time. Since the second harmonic has twice the frequency of the fundamental, sounding one octave higher, the situation is comparable to that of the C' and C'' tuning forks sounded simultaneously. Take care not to aim the open end of the pipe directly at the microphone. While it is not the purpose here to enter into a discussion of the physics of why the pipe produces tue second harmonic as well as the fundamental, the trigonometry of the situation lends itself well to the discussion of combining wave forms. The result of the first and second harmonics sounding together is the $y = \text{sine } x + \text{sine } 2x$ situation, with the wave form on the 'scope matching quite closely the wave form predicted graphically.

Extending the concept of combining wave forms a bit further, it can be pointed out that under certain circumstances two maxima or two minima of approximately the same amplitude will occur at the same time, resulting in an amplitude peak which stands out from the surrounding portions of the compound curve. In physics these peaks are called re-enforcements. Similarly, if a maximum and a minimum occur at the same time, partial or even complete cancellation may be produced.

In sound, these peaks, or re-enforcements, represent points of increased amplitude known as beats. If the two frequencies sounding simultaneously are close together, the beats become audibly distinguishable. This can be easily demonstrated with matched resonance boxes and tuning forks, one of which has one prong "loaded" with a short piece of rubber tubing, altering its mass and thereby reducing its frequency to slightly below that of the other fork.

From a trigonometric standpoint, the method of adding ordinates will show how the peaks occur. (This is a tedious process and the graph should be prepared in advance in order to save class time.) The use of the microphone and 'scope setup presents a graphic display of the momentary increase in amplitude as the beats occur, adding authority to the prepared graph. Using a sweep frequency about double that of the forks employed, the 'scope will show the fluctuating amplitude, although the trace cannot be stopped as in former cases.

An interesting side discussion may be found in the trigonometric proof that the number of beats per second is equal to the difference

of the two beating frequencies in cycles per second. This proof is not at all beyond the better high school trigonometry student. Discussions of the proof may be found in many college physics texts.[1]

Further applications are found in the beat frequency oscillator and the heterodyne principle employed in many radio receiving and transmitting sets. A suitable reference text is listed in the bibliography.

The compounding of curves of periodic functions produces other interesting and useful wave forms. Since any periodic function can be expressed as the sum of a number of sine or cosine functions, the component wave forms of which a complex function is composed can be computed using the Fourier series method if its equation is known, or found by the use of an instrument known as a "harmonic analyzer" if its graph is available. An example of such a series, which produces the "square" wave form used frequently in electronics, is:

$$y = A \text{ sine } x + 1/3A \text{ sine } 3x + 1/5A \text{ sine } 5x + \cdots.$$

See figure 3, a and b. A second such series is that which produces the familiar "sawtooth" wave form:

$$y = A \text{ sine } x + 1/2A \text{ sine } 2x + 1/3A \text{ sine } 3x + \cdots.$$

See Figure 3, c and d. It is this sawtooth wave form which is employed to provide the voltage changes, described previously, in the internal sweep signal of the oscilloscope.

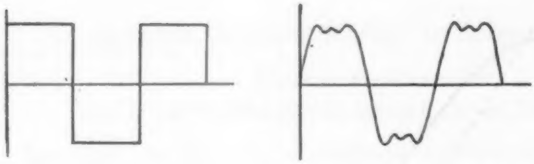
If a suitable generator is available, these two wave forms are easily shown on the 'scope. Such generators are available commercially in kit form at reasonable cost.

Equipment

A wide range of suitable equipment is available for this type of demonstration. The equipment used in this demonstration includes a variable impedance microphone (Shure model 51), a twenty watt audio amplifier (Heath model A9-C), a five-inch general purpose oscilloscope (EICO model 425), and a small speaker and enclosure, in addition to the tuning forks, resonance boxes and organ pipe previously mentioned.

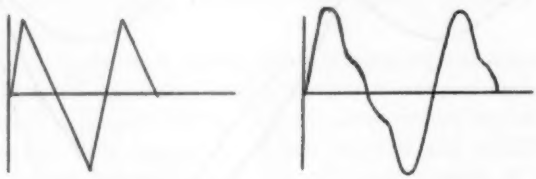
The equipment need not be expensive, since many adequate kits for construction of the electronic components are available at reasonable cost, if the school does not already possess such items as an oscilloscope. The equipment pictured (excluding the tuning forks, resonance boxes and organ pipe) involved a total cost of approximately $125.00, spread over a period of two years. A larger 'scope, such at one of the seven-inch models now on the market, would be a distinct advantage from the standpoint of classroom visibility, but the five-

[1] For example see: Sears, F. W., *Principles of Physics*, Vol. I (Mechanics, Heat and Sound), Addison-Wesley, 1947, pp. 500–502.



a. Idealized square wave form

b. Sum of first three terms of Fourier series
$y = \text{sine } x + 1/3 \text{ sine } 3x + 1/5 \text{ sine } 5x$



c. Idealized sawtooth wave form

d. Sum of first three terms of Fourier series
$y = \text{sine } x + 1/2 \text{ sine } 2x + 1/3 \text{ sine } 3x$

FIG. 3

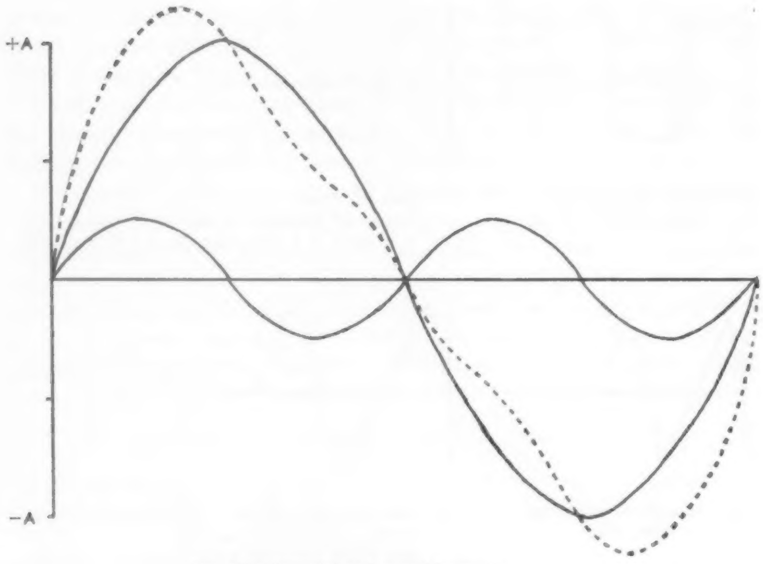
inch model pictured has been used successfully with groups as large as twenty students. An interesting development for classroom work might be the conversion of a television set for use as an oscilloscope, along the lines indicated in the magazine reference cited in the bibliography.

In an early presentation of this series of demonstrations, the school's sixteen millimeter motion picture projector was employed as an audio amplifier in conjunction with the oscilloscope. Many such projectors have a microphone input. Since they are intended for speech amplification they are limited as to the amount of gain they can provide without clipping the peaks.

The addition of a sine and square wave generator might be considered as a worthwhile extension. A simple sawtooth wave generator can be constructed following the pattern indicated in the reference listed in the bibliography.

TIME AND SEQUENCE

Naturally the individual instructor will have his own ideas about where the graphic representation of trigonometric expressions fits into the sequence of the trigonometry course. The importance of graphing technique makes the slight extra time required for presenting demonstrations of this sort well spent. Two average class periods should cover the demonstration time, inserted into the coverage



Sum of fundamental and second harmonic.
(Sum indicated in dotted line)
$y = A \text{ sine } x + \frac{1}{4}A \text{ sine } 2x$

FIG. 4

of graphs of functions as individual opinion dictates. The approximate order in which the demonstrations are presented here should probably be maintained, however.

The writer feels that the time required for setting up equipment and rehearsal has been more than paid for in the reaction of classes to which the demonstrations have been presented. It may well be that, under the pressure of a tight time schedule, several periods devoted to work of this type would be worth the sacrifice of some time usually dedicated to prolonged practice in the solution of the sacred right triangle!

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3. SEARS, F. W., *Principles of Physics*, Vol. I, Part 3, "Sound," Addison-Wesley, 1947.
4. SUTTON, R. M., *Demonstration Experiments in Physics*, McGraw-Hill, 1938.

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Seventh Graders' Ability to Solve Problems

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Are there characteristic differences between high and low achievers in seventh grade problem solving? Do they differ significantly in various types of reading skills? What quantitative skills are necessary for successful achievement in problem solving? Are any specific mental abilities closely associated with problem solving? Does socio-economic status influence the seventh grader's success in problem solving?

A study was made recently to seek answers to such questions. Pairs of high and low achievers in seventh grade problem solving matched according to sex, IQ, and mental age in months were compared with respect to certain factors and abilities. The measure of problem solving ability was the pupil's score on a standardized test. The results of the study revealed several significant facts which may be useful to the teacher in planning instruction to help seventh graders improve their ability to solve verbal problems. The details of the study are not given here. However, some of the general conclusions are presented with comments about their educational implications.

Characteristic Differences Between High and Low Achievers in Problem Solving

The areas in which significant differences were found in favor of high achievers are summarized in the upper part of the outline below. In the second section of the outline are the areas in which there were no significant differences. The areas in the third section show differences which were in favor of the low achievers in problem solving. All of the differences listed as significant proved to be statistically significant at or above the 5% level of confidence.

Differences Between High and Low Achievers In Problem Solving

A. Significant differences in favor of high achievers
   1. Specific mental abilities
      a. General reasoning ability
      b. Ability to understand verbal concepts
   2. Quantitative skills
      a. Understanding mathematical terms and concepts
      b. Skill in computation
   3. General reading skills
      a. Comprehension of reading materials
      b. Understanding words in context
   4. Problem solving reading skills

    a. Comprehension of statements in problems.
    b. Selection of relevant details in problems
    c. Selection of correct procedures to solve problems
  5. Interpretation of quantitative materials
    a. Finding data from graphs, tables, charts, and maps
    b. Perception of relationships involving comparison of data
    c. Recognition of limitations of given data

B. No significant differences
  1. Specific mental abilities
    a. Ability to use words easily
    b. Ability to visualize objects in two or three dimensions
  2. Socio-economic status
  3. Quantitative skills
    a. Timed addition of whole numbers

C. Significant differences in favor of low achievers
  1. Interpretation of data errors
    a. Tendency to require more information than necessary to judge data
    b. Going beyond the data given
    c. Inaccuracy due to carelessness, reading difficulties, or inability to see relationships

Conclusions

Several general conclusions derived from the foregoing differences seem to be justified. They are stated as follows.

Mental ability. Two specific mental abilities are essential to the seventh grader's ability to solve arithmetic problems. His ability to solve logical problems through reasoning and his ability to understand words or verbal concepts are strongly associated with his ability to solve arithmetic problems. In contrast to the second ability, the ease with which the seventh grader uses words has little bearing on his success in problem solving.

Socio-economic status. Socio-economic status proved to have little influence on the seventh grader's ability to solve arithmetic problems. It is conceivable, however, that a different conclusion might have been reached if a more refined measure had been employed to determine socio-economic status.

Quantitative skills. The problem of solving ability of seventh graders seems to depend in large measure upon their ability to (1) interpret and use the number system, (2) employ the fundamental processes with various types of numbers, and (3) comprehend and use concepts of measurement. Accuracy in rapid addition of whole numbers alone, however, is insufficient to insure success in problem solving. This conclusion is in sharp contrast to the second item stated above.

Reading skills. High achievers in seventh grade problem solving are superior in their ability to read and comprehend written materials which pupils are usually called upon to read in school. They are able to (1) understand the theme of a reading selection, (2) understand the main idea of a paragraph, (3) infer logical ideas which are

implied, (4) grasp small details in written materials, and (5) understand the meaning of words in context. High achievers are superior also in reading skills which tend to be special to the analysis of verbal arithmetic problems.

Interpretation of quantitative materials. High achievers in seventh grade problem solving are superior to low achievers in their ability to visualize and interpret quantitative facts and relationships. They are less overcautious in judging data, go beyond the data less often when there is insufficient data to justify a decision, and make fewer errors due to carelessness, reading difficulties, or inability to identify relationships in given data.

Guides to Planning Instruction in Problem Solving

A number of factors in this study proved to be closely related to the problem solving ability of seventh graders. In light of these factors, several educational implications seem to be justified. The following implications are recommended as guides which may help the teacher plan a sound instructional program in problem solving.

A differentiated program. The fact that there are important differences between high and low achievers in problem solving shows that a differentiated program is needed in arithmetic problem solving. Differences in areas such as mental ability, reading ability, and quantitative skills indicate that instruction in problem solving should be adapted to the needs and abilities of the pupils. There should be some problems with which all seventh graders can be successful. There should be some problems which challenge more capable pupils. Diagnosis of difficulties and activities to improve problem solving skills, instructional materials, learning experiences, and goals to be achieved should be selected and organized to meet individual variations in problem solving ability.

Selection of printed materials. Since reading ability is related to problem solving, written materials should be selected carefully. The vocabulary and language structure of selected materials should be appropriate to the reading level of pupils who are to use them.

Improvement of general reading ability. Seventh graders may be helped through reading instruction to improve their ability to solve verbal problems. Attention should be given to developing a basic vocabulary and fundamental reading skills similar to those required in other subjects if weaknesses exist in the pupil's general reading ability.

Development of reading skills related to problem solving. Specific reading skills fundamental to reading and solving arithmetic prob-

lems need to be systematically developed. Ample opportunities should be provided for the seventh grader to develop the ability to comprehend the meaning of items and statements contained in verbal problems. They also need to develop the ability to recognize verbal clues required in the solution of problems, such as how much more, total cost, equal monthly payments, etc. Practice in reading skills related to the selection of the correct process needed to solve the problem should be provided for the seventh grader. An understanding of basic and enriched meanings of mathematical terms and concepts may be developed through vocabulary activities similar to those used in other subject areas.

Development of mathematical concepts. Instructional practices in problem solving should promote the systematic development of the ability to understand mathematical concepts and relationships. Seventh graders should have many opportunities to discover relationships between quantities and processes. They need to solve many meaningful problems in which mathematical concepts and relationships are involved. Seventh graders need to be able to read and interpret numbers and to understand the decimal nature of the number system. They also need to develop such abilities as (1) understanding the meaning of fractional parts, (2) skill in using various measuring devices, and (3) understanding concepts of size, shape, form, linear measure, time, speed, weight, capacity, temperature, etc.

Skill in fundamental operations. To be successful in problem solving, seventh graders need the ability to employ the fundamental processes with understanding. They need to develop fluency in the use of the four basic operations with whole numbers, fractions, decimals, per cent, denominate numbers, formulas, and simple equations. Accuracy in the use of the four processes with various types of numbers should be developed before speed in computation is emphasized or expected.

Interpretation of quantitative materials. Development of the ability to interpret quantitative materials should be an integral part of instruction in problem solving. Seventh graders need many opportunities to visualize and interpret facts and relationships in charts, tables, graphs, and maps. They need to be able to make comparisons with data, to recognize limitations of given data, and to discriminate between relevant and irrelevant detail.

The amount of radiation needed to make a complete survey of the mouth by X-ray is below the detectable damage level.

The Journal of the American Dental Association said, "The routine use of modern X-ray equipment and techniques for dental diagnostic purposes is not harmful."

Why Ninth Grade General Science?

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There is a constant sniping at the ninth grade general science program by critics who are not aware of its real importance in the total educational program. Various critics suggest that the final year of general science be placed in the eighth grade and general biology be moved down to the ninth grade. Others suggest that general science be changed to an activities program stressing scientific method instead of content, or that part of the course be assigned to health courses, or that it be made elective or required only for slow learners. Some objectors even advocate unifying science and mathematics or social studies.

There are several sound reasons for having a full year of ninth grade general science required for all pupils.

General science is the terminal physics course for far more than half of all high school pupils, and is terminal for all drop-outs. The same applies, to a less important degree, to chemistry. Surely no citizen should face the atomic and space age with only the amount of physics and chemistry learned by the end of the eighth grade!

Moreover, some modern physics courses, such as the one developed under direction of the National Science Foundation, largely dispense with descriptive physical science in favor of development of theory. Even superior pupils who enter these courses need knowledge of fundamentals of how the world works before they try to figure out why it works. General science provides the know-how of common things.

A good ninth grade general science course summarizes, organizes, and completes the nine-year program of largely descriptive science. This is possible because of the increased ability of ninth grade pupils to generalize. That is, as expressed by one pupil, "I didn't learn so much that is new this year, but I think that now I understand what I thought I knew." It is unlikely that the critical age for much real understanding of major science principles is below fourteen years.

The range of general science units through several basic sciences is of great value in meeting individual needs and in stimulating interest in some field of science. Pupils who are completely bored by one unit often are stimulated to intense activity by another from a different field. Thus, during the school year, pupils may become interested in such special outside activities as studying tumors in rats, building a ham radio receiver, collecting minerals, computing Mendel's law as a

special science-mathematics project, making a turbine from tin cans, building an electrostatic generator, and working on a personal dieting program for figure control. These, of course, are not typical class activities, but are developments of special interests aroused in class and club work. The exploratory value of general science exists now as it did when the course originated.

The changing pattern of general science units meets young-adolescent needs for change and variety. The attention span of a fourteen year old pupil is much less than an adult's. Thus pupils who become tired of studying electricity can be remotivated later in the school year by changing the unit name and approach to "electronics."

General science is not a direct repetition of elementary science. Repetition should be planned, and not encountered at random. Taking the magnetism experiment as the one most overworked in elementary science, a completely fresh approach is needed in ninth grade. Use of an alternating current magnetizing device which blows a fuse provides an entirely new observational opportunity. Yet in testing magnetized pieces of steel, the pupil reviews all previously learned information about magnets. The difference is is that the pupil, while reviewing magnetism, is also learning about induction. Induction is rarely taught directly in any elementary science class.

The trouble caused by repetition in general science is its routine misuse. It is doubtful that there is enough effective repetition of factual information, as is indicated by studies showing that facts are too quickly forgotten. Repetition of facts is properly used in two ways. They may be applied to help in understanding new and interesting situations. They can be used to develop and solve problems. The most frequent cause of failure of activity learning programs is the lack of factual information by the pupil, who is unable to understand, state, and organize an attack upon a simple problem. Activity without problem solving is mostly mere busy work.

It must be recognized, of course, that general science does not exist alone in the so-called 12-year science program. Better coordination of science courses at all grade levels certainly is needed. Some of the fault is that too much is attempted in elementary sciences. In the lower grades much more emphasis should be placed upon both experiencing and experimental activities. Too often high school science is pushed to lower and lower grade levels, diluted, and made more and more meaningless as it is moved downward. For example, it is doubtful that any real rocket science, except a simple statement of the law of reaction, can be taught effectively below the high school level. Yet almost every grade school pupil is frequently exposed to a sort of quack rocket science, both in school and out.

In coordinating elementary science with upper grades, children

should learn and experience as children, and not as immature adults.

General science in a properly organized program, of all science courses, best meets the needs of individual ninth grade pupils. Over some 50 years by trial and error, controlled research, and intelligent observation a ninth grade science course has evolved. Subject to constant re-evaluation and revision, this course is suited to meet the needs of average and superior ninth grade pupils. The course is fairly widely accepted by curriculum makers and writers of textbooks.

This ninth grade course is not well fitted to the needs of slow learning ninth grade pupils nor to average seventh and eighth grade pupils. The mental maturity of an average 13-year old is about 93 per cent of that of an average 14-year old. The mental maturity of the average 12-year old seventh grader is only about 85 per cent of that of the typical ninth grader.

It is generally accepted that mental maturity scores of less than 95 indicate need for special kinds of science and mathematics on the ninth grade level. This special kind of science is not the typical ninth grade course, for it must be reduced in difficulty by eliminating much abstract content and by reducing emphasis on reasoning.

A nearly ideal plan of class organization is to divide groups of 100 to 120 typical pupils into two average classes, one slow-learning class, and one somewhat accelerated class. (It is very important not to place the bright, emotional deviate in the slow-learning class.) Such grouping is rarely attempted, and where it is, teachers usually lack time, content material, and equipment to produce much difference in the way various level classes are taught. Teachers should plan with counselors and administrators to arrange proper ability grouping and to provide time and materials to make it operate well.

There is now, where conditions and personnel permit, much good science teaching on the ninth grade level. Some of the conditions typical of such good teaching and learning are listed briefly:

1. One or two basic textbooks, or a learning guide with textbook references, provide basic coverage of fundamental facts.
2. Many visual aids are used, particularly motion pictures closely coordinated with other class activity. Models and diagram-type charts are available.
3. Simple experimental equipment, microscopes, and an equipped area for growing plants are available for use by some pupils. Work areas are out of hearing and line-of sight of pupils doing routine work. Pupils who have reason to solve problems by use of equipment are given time to do so.
4. The teacher has conveniently at hand a variety of demonstration equipment which is used to stress understanding of basic principles.

5. Notebooks or workbooks are used to record essential facts and observations as concisely and briefly as possible. Stress on spelling, vocabulary, and accuracy of statements is constant. "Pencil pushing busy work," made-work drawings, and lengthy summarizing are not used.

6. Mathematics is used in such appropriate areas as study of heat, mechanics, and electricity, and to some extent in such areas as elementary chemistry and heredity. This activity is reserved for above-average pupils.

7. The science class program is coordinated with a club program, in-school or extra-curricular, which gives added direction to out-of-school activities.

8. Finally, the entire science program stresses learning principles of science by observation, by systematic reading, by organizing random information, by application, and by memorization of basic principles.

Before school curriculum making groups plan to abolish ninth grade general science, they should first examine causes for discontent with the present course. They will probably find nothing wrong with the basic philosophy of the course. Instead, possible dissatisfaction may arise from poor course organization, poor teaching methods, inadequate facilities, overloading and overcrowding of classes, lack of teacher preparation time, unwise selection of texts and learning helps, too great a range of pupil ability in every class, or—unhappily in some schools—most or all of these deficiencies. Substituting a course with a new name or moving a high school science course into ninth grade will not solve problems of a failure to meet pupil needs adequately.

FARM CHEMICALS PROVE HAZARD IN CALIFORNIA

Farm chemicals have harmed hundreds of persons in California, state health officials reported.

As the use of insecticides, chemical fertilizers and soil additives has increased, so has the number of reports of occupational disease attributed to these agents.

There were 749 reports of disease attributed to the chemicals in 1957. Nearly a third of these were attributed to organic phosphate pesticides.

These are among the most hazardous materials used as pesticides. Organic phosphate chemicals may enter the body directly through the skin, as well as by inhalation and swallowing.

Since many persons find the concept of poisoning through the skin hard to understand, workers often fail to wear protective clothing when applying such chemicals and to wash themselves thoroughly afterward.

Other pesticides contain nicotine, fluorides and arsenic compounds, any of which can be hazardous. California is estimated to use one-fifth of the total quantity of pesticides in the U. S. One death attributed to agricultural poisoning was recorded in 1957.

An Annotated Bibliography for Teachers of Mathematically Gifted High School Students

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The following bibliography is intended to help the teacher of the mathematically gifted student to find out what should be taught and how the gifted student can be challenged. It deals with two phases of teaching the mathematically able student by including articles, pamphlets and books expressing 1) ideas on how to teach the gifted student, and 2) what materials should be taught.

The bibliography is divided into two parts. Part One contains source material which can be helpful to the teacher in preparing to teach the more able classes in mathematics. Part Two contains a list of books which should be challenging to the capable reader.

I. Material Helpful to the Teacher

Adkins, Jackson B., "College Prep Mathematics in Grades Nine through Twelve, "*The Mathematics Teacher*, 50: 209–13, March 1957.

A college prep course is described by showing how the use of four major mathematical ideas unifies the program of grades nine through twelve.

Ahendt, M. H., "Education of the Mathematically Gifted," *Phi Delta* Kappan, 34: 285–87, April 1953.

Discusses ability of gifted students, desirability for their early identification, qualification of teachers of gifted students and administrative policies which may help the gifted program.

———, "Mathematics and Science," *National Education Association Journal*, 46: 109–110, February 1957.

Gives results of study of 1953 and 1956 winners in the Annual National Honor Society Program. Shows top students do not avoid hard subjects and that they enjoy work in mathematics and science, since 58% of total group of 620 indicated they planned to spend their lives in scientific or technical work.

Albers, Mary Elizabeth, and May V. Seagoe, "Enrichment for Superior Students in Algebra Class," *Journal of Educational Research*, 40: 487–95, March, 1947.

An attempt was made to test the desirability of providing an enrichment unit for superior students in second semester ninth grade algebra classes. Conclusions point to a saving of time, as well as an interest sufficiently great to provide the necessary motivation.

Baumgartner, Reuben A., "A Mathematics Curriculum for the Gifted," School Science and Mathematics, 53: 207–12, March, 1953.

Here are listed several suggested courses and topics which are or may be taught to the mathematically gifted.

———, "The Status of the Secondary Mathematics Program for the Talented," *The Mathematics Teacher*, 49: 535–40, November, 1956.

There is an increasing interest in revising the mathematics program to take care more adequately of the talented high school pupil.

Begle, E. G., "The School Mathematics Study Group," *National Association of Secondary School Principals Bulletin*, 43: 26–31, May, 1959.

The program of the School Mathematics Study Group is studied and their projects are listed in brief.

Brinkmann, H. W., "Mathematics in the Secondary Schools for the Exceptional

Student," *The American Mathematical Monthly*, 54: 319–23, May, 1954.
Main ideas behind the mentioned plan are to break down the standard compartmentalized program, to introduce new subject matter, and at the same time suggest the elimination of certain traditional items.

Brown, Kenneth E., "Mathematics a Key to Manpower," *School Life*, 36: 26–27, November 1953.
Gives statistics on number and percentage of pupils in mathematics in the last four years of high school from 1890 to 1953. Points out way to increase supply of scientists and mathematicians.

Brown, Kenneth E., and Philip G. Johnson, *Education for the Talented in Mathematics and Science*, Bulletin, U. S. Department of Health and Education and Welfare. Office of Education, 1952, 34 pp.
What has been done and what can I do to help the gifted student in my own teaching position?

Brueckner, Leo J., F. E. Grossnickle, and John Reckzeh, *Developing Mathematical Understandings in the Upper Grades*. Philadelphia: J. C. Winston Company, 1957. pages 502–548.
The specific chapter deals with the need and objectives for enrichment and it also includes topics to be discussed in the classroom.

Brumfiel, Charles, "The Foundations of Algebra" *The Mathematics Teacher*, 50: 488–92, November, 1957.
This article discusses topics which should be introduced and used in an accelerated Algebra course.

Brydegaard, Marguerite, "Creative Teaching Points the Way to Help the Brighter Child in Mathematics," *The Arithmetic Teacher*, 1: 21–24, February, 1954.
Recognizing and challenging pupils to do creative thinking is a method of teaching used by good teachers the world over.

Burr, Irving W., "What Principles and Applications of Statistics Should be Taught in High School and Junior College?" *The Mathematics Teacher*, 44: 10–12, January 1951.
Suggestions are made for developing certain concepts of statistics.

Butler, Charles H., and F. Lynwood Wren, *The Teaching of Secondary Mathematics*. New York: McGraw-Hill Book Company, Inc., 1951. Pages 339–74 and 508–41.
These chapters deal with further topics to be considered in Algebra and the possibility of teaching Calculus.

Carlson, Edith Fox, "Project for Gifted Children: A Psychological Evaluation," *American Journal of Orthopsychiatry*, 15: 648–61, October 1945.
Gives results of case studies of 25 gifted children, following 16 of them for four years through the seventh grade. Shows results of placing children in special classes and giving them a less constraining environment in the classroom. Points up importance on continually challenging the gifted student.

Carnahan, Walter H., Editor, *Mathematics Clubs in High Schools*, National Council of Teachers of Mathematics, 1958.
What can Mathematics clubs do to increase the knowledge of able Mathematics students?

Cherry, W. J., "Methods of Selecting Freshmen for Accelerated Work in Mathematics," SCHOOL SCIENCE AND MATHEMATICS, 58: 467–71, June, 1958.
A report on the results found in grouping the mathematically gifted in one class.

Clark, N., "Challenge to the Gifted," *Mathematics Teacher*, 48: 434–5, October 1955.
Gives another viewpoint on the "honors program" at Kirkland, Washington (See Clark, N., and D. Lander). Indicates courses offered are non-traditional.

———, and D. Lander, "Honors Program," *Mathematics Teacher*, 48: 371, May 1955.

Reports on "honors program" at high school in Kirkland, Washington. Program extends from twelfth grade down through tenth grade. Included are such topics as number theory, number systems, the algebraic theory of sets, geometrical constructions, projective, axiomatic, and non-Euclidean geometries. Gives textbooks used.

College Entrance Examination Board, Commission on Mathematics. *Appendices*. New York, 1959. 223 pp.

A fine resume of the content of courses recommended to be taught in high school mathematics classes.

College Entrance Examination Board, Commission on Mathematics. *Program for College Preparatory Mathematics*. New York. 1959. 62 pp.

This report gives a brief outline of courses to be taught in high school mathematics. It describes vital role of the teacher.

Conant, James B., *The Identification and Education of the Academically Talented Student in the American Secondary School*. National Education Association, February 1958, pages 97–103.

The curriculum and education of the academically talented in Mathematics is discussed.

Cornog, William H., "The High School Can Educate the Exceptionally Able Student," *National Association of Secondary-School Principals*, 39: 380–86, April 1955.

Discusses study of admission of exceptional student into college with advanced standing based on passing of certain tests.

Cunningham, Harry A., "Some Challenging Problems in Teaching High School Science to Gifted Children," SCHOOL SCIENCE AND MATHEMATICS, 52: 373–380, May 1952.

Makes many recommendations applicable to both mathematics and science. Has short bibliography relative to science mainly.

Cutts, Norman E., and Nicholas Moseley, *Teaching the Bright and Gifted*. New Jersey: Prentice-Hall, Inc., 1957. 260 pp.

This book has been designed to give practical help to classroom teachers in elementary and secondary schools. It should help to identify and understand the gifted student.

Denbow, Carl H., "To Teach Modern Algebra," *The Mathematics Teacher*, 52: 162–170, March, 1959.

This article states that we don't need to choose between modern and traditional mathematics. The author shows that they blend together quite naturally.

Douglass, Earl R., "Issues in Elementary and Secondary School Mathematics," *Mathematics Teacher*, 46: 290–94, May 1954.

Discusses important issues such as grouping, grade level of algebra and geometry, and homework. Has several ideas helpful to teacher of gifted students.

Edwards, P. D., P. S. Jones, and Bruce E. Meserve, "Mathematical Preparation for College," *Mathematics Teacher*, 45: 321–330, May 1952.

Lists important major fields in college and the preparation necessary for each field to be obtained in high school mathematics courses. Gives good information to student planning college work and procedure for proper preparation.

Elder, Florence L., "Providing for the Student with High Mathematical Potential," *The Mathematics Teacher*, 50: 502–6, November, 1957.

An article on identifying the mathematically gifted student, and also some topics that should be taught are discussed. What are the reactions of the parents of these gifted students?

Eves, Howard, and Carroll V. Newsom, *An Introduction to the Foundations and Fundamental Concepts of Mathematics*. New York: Rinehart and Company, 1958. 347 pp.

This is a good reference book to prepare a teacher to understand some

of the modern mathematical concepts. This book includes an excellent section on set theory.

Fehr, Howard F., "General Ways to Identify Students with Scientific and Mathematical Potential," *The Mathematics Teacher*, 46: 230–34, April 1953.

The task of identifying giftedness is not an easy one and there are some approaches and traits which are studied in this article.

———, "Mathematics for the Gifted," *National Association of Secondary School Principals Bulletin*, 38: 103–10, May, 1954.

A discussion of the motivation, identification, and curriculum provisions for the mathematically gifted.

———, *Secondary Mathematics, A Functional Approach for Teachers*. Boston: D. C. Heath and Company, 1951. 405 pp.

This book discusses many topics which would be ideal for the gifted student to learn about. This would be a good supplementary text for any senior high school teacher.

Fehr, Howard F., "The Goal is Mathematics for all," SCHOOL SCIENCE AND MATHEMATICS, 56: 109–20, February, 1956.

This discussion gives indication of a reorientation in Mathematics education. Changes are appearing in both teaching procedures and subject matter content that promise a richer and more enduring learning experience.

Freese, F., "Gifted Students in Senior High School Mathematics," *National Association of Secondary-School Principals Bulletin*, 43: 71–4, May 1959.

Cites two ways of working with gifted students: in homogeneous groups and individually in regular non-segregated groups. States need for recognition of superior students. Gives several criteria for admission to special classes and some activities possible.

Freitag, Herta T. and Arthur H., "Using the History of Mathematics in Teaching on the Secondary School Level," *The Mathematics Teacher*, 50: 220–24, March, 1957.

The historical approach to the teaching of mathematics requires that mathematics be presented as an ever-envolving human endeavor.

Gager, William A., "The Functional Approach to Elementary and Secondary Mathematics," *The Mathematics Teacher*, 50: 30–34, January, 1957.

The big objective in the functional type of teaching in mathematics is to make contact with the personal experiences of the pupils in such a way that they will see some value in what is to be done and learn something that is worthwhile.

Gordon, Garford G., *Providing for Outstanding Science and Mathematics Students*, California: University of Southern California Press, 1955. 100 pp.

This book deals with the problems and methods of dealing with the gifted student. It gives suggestions of several types, depending upon the size of the school.

Hankins, Donald D., Jr., "A Tentative Guide for a Twelfth Grade Honors Course," San Diego City Schools, 1955.

This pamphlet deals with material to be taught to twelfth grade students in a Mathematics honors course.

———, "Realignment of High School Mathematics Programs," San Diego City Schools, 1956.

This pamphlet contains outlines of Advanced Algebra and Geometry for students interested in liberal arts and Geometry for students interested in Mathematics and Science.

Hartung, Maurice L., "High School Algebra for Bright Students, "*The Mathematics Teacher*, 46: 316–321, May, 1953.

A description of the sort of algebra course that should be offered to students, both college bound and not college bound.

Hetland, Melvin, and Harold Glenn, "A Program for the Mathematically Gifted," *California Journal of Secondary Education*, 32: 334–37, October 1957.

Reports on experimental program in the Long Beach Public Schools, offering Algebra I and Plane Geometry in half the usual time. Covers selection, teaching, evaluation and elimination procedures. Closes with prerequisites for success of such a program.

Irwin, Lee, "The Organization of Instruction in Arithmetic and Basic Mathematics, in Selected Secondary Schools," *Mathematics Teacher*, 46: 235–40, April 1953.

Gives report on study to discover practices useful in teaching nontraditional mathematics and to make recommendations for reorganization of high school mathematics program. Twelve recommendations are made.

Jackson, H. O., "Superior Pupil in Mathematics," *Mathematics Teacher*, 52: 389–417, May 1959.

Johnson, Donovan A., "Let's do Something for the Gifted in Mathematics," *The Mathematics Teacher*, 46: 322–25, May, 1953.

This article points out the need for helping the gifted. The author also gives suggestions as to what might be done in your high school along the same line.

Kelly, Inez, "Challenging the Gifted Student," *School Life*, 35: 27–28, November, 1952.

This article describes what one Mathematics teacher does to challenge the more able students in the field of mathematics. An interesting Mathematics club is explained.

Kennedy, Joe, "Modern Mathematics in the Twelfth Grade, "*The Mathematics Teacher*, 52: 97–100, February, 1959.

A description of an experimental twelfth-grade mathematics course taught by the author in a Wisconsin school.

Kieffer, M., "Meeting the Needs of Cincinnati's Gifted Pupils in Mathematics," *National Association of Secondary-School Principals Bulletin*, 43: 89–92, May 1959.

Traces method for advanced placement in high schools in Cincinnati. Gives evaluation of program as conducted with outline of courses of study.

Kinney, L. B., and C. R. Purdy, *Teaching Mathematics in the Secondary School*. New York: Rinehart and Company, Inc., 1952, pages 136–187.

The chapter deals with various topics which should be covered in an advanced mathematics course. A bibliography at the end of the chapter has helpful information.

Kraft, Ona, "Providing a Challenging Program in Science and Mathematics for Pupils of Superior Mental Ability," SCHOOL SCIENCE AND MATHEMATICS, 52: 143–47, February, 1952.

An attempt is made to call the readers attention to some of the methods and materials that are especially appropriate for teaching the able.

Laframboise, Marc A., "The Training of Mathematics Teachers," SCHOOL SCIENCE AND MATHEMATICS, 55: 389–92, May, 1955.

Academic qualifications for Mathematics teachers are discussed.

———, "College Mathematics Teacher," SCHOOL SCIENCE AND MATHEMATICS, 58: 108–110, February 1958.

Is follow-up to previous article. Outlines quite well his recommended program for a student getting a master's degree in mathematics with intent to teach. Usable to show gifted student extent of preparation necessary for college teaching.

Langer, R. E., "Time is Running Out," *Mathematics Teacher*, 49: 418–24, October 1956.

Points up increasing importance of mathematics and shows need for turning out students prepared better mathematically from high school. Compares U.S. curriculum with that of U.S.S.R.

Levy, N., "Toward Discovery and Creativity," *Mathematics Teacher*, 50: 19–22, January 1957.

Relates instances where students using individual methods have "dis-

covered" mathematical relationships and how, when properly guided, gifted students experience a feeling of adventure in mathematics and thus are strongly motivated.

Latino, Joseph J., "An Algebra Program for the Bright Ninth Grader," *The Mathematics Teacher*, 49: 179–84, March, 1956.

Here are listed some suggestions as to what should be done about the bright student in the mathematics class.

Lloyd. Daniel, B., "Ultra-Curricular Stimulation for the Superior Student," *The Mathematics Teacher*, 46: 487–89, November, 1953.

The author makes practical suggestions for meeting the needs of superior pupils in mathematics.

MacDuffee C. C., "What Mathematics Shall we Teach in the Fourth Year of High School?" *The Mathematics Teacher* 45: 1–5, January 1952.

It is recommended that we include introductory parts of Analytical Geometry and Calculus in high-school courses.

McCoy. M. Eleanor, "A Secondary School Mathematics Program," *National Association of Secondary School Principals Bulletin*, 43: 12–18. May 1959.

The article discusses the University of Illinois experimental mathematics program. It includes the general topics that are covered.

MacLane, Saunders, "The Impact of Modern Mathematics," *National Association of Secondary-School Principals Bulletin*, 38: 66–70, May 1954.

Shows need for changing of curriculum to include aspects of "modern mathematics." Gives examples where inclusions may be made in algebra, geometry and trigonometry.

McWilliams, Earl M., "The Gifted Child in the High School," *National Association of Secondary-School Principals Bulletin*, 39: 1–9, May 1955.

Meder, Albert E., Jr., "Modern Mathematics and its Place in the Secondary School," *The Mathematics Teacher*, 50: 418–23, October 1957.

How should modern mathematics be introduced in the present mathematics program? Can it help clear the more capable students' mind of some questions?

———, "Proposals of the Commission on Mathematics of the College Entrance Examination Board," *National Association of Secondary School Principals Bulletin*, 43: 19–26, May 1959.

A short review of the Commission on Mathematics of the College Entrance Examination Board and the work they are trying to accomplish.

Merriel, D. M., "College Mathematics in the High School," *Mathematics Teacher*, 51: 556–7, November 1958.

Author gives own opinion as to desirability of introducing analytic geometry and calculus in the high school. Cites several alternatives to such inclusions, such as using the Illinois study and giving a course in the theory of equations.

Meserve, Bruce E., "Topology for Secondary Schools," *The Mathematics Teacher*, 46: 465–74, November 1953.

Illustrates some of the properties of topology and suggests their use in interesting the superior pupils in mathematics and in emphasizing the importance of this fundamental mathematical concept.

Moore, Lillian, "The Challenge of the Bright Pupil," *The Mathematics Teacher*, 34: 155–57, April 1941.

A brief discussion of the general lack of provision for the bright pupil, especially in mathematics, with suggestions for enriching the work for his benefit.

National Education Association, Research Division, "High School Methods with Superior Students," *Research Bulletin*, 19: 191–193, September 1941.

Is part of exhaustive study on superior students. Gives examples of supplementary teaching materials, varied assignments and teaching methods, and enrichment activities in special mathematics classes. Also reports a continuing teacher plan and student assistantships. Old but informative.

Northrop, E. P., "Modern Mathematics and the Secondary School Curriculum," *The Mathematics Teacher*, *48*: 386–93, October 1955.

This article underlines the current dissatisfaction with the high school curriculum and offers a few pointers for the next steps to be taken.

Norton, Monte S., "Enrichment as a Provision for the Gifted in Mathematics," SCHOOL SCIENCE AND MATHEMATICS, 57: 339–45, May 1957.

The true enrichment process does not include merely the addition of extra work for the pupil concerned, but aims to enrich the pupils' entire learning process.

———, "What are Some of the Important Factors to Consider in a Program of Identifying the Gifted Pupil in Science and Mathematics," SCHOOL SCIENCE AND MATHEMATICS, 57: 103–8, February 1957.

This article is an accumulation of the latest thinking of authorities in the field in regard to important considerations schools should make in administering an identification program in science and mathematics.

Passow, A. H., and D. J. Brooks, Jr., "Mathematics and the Gifted Student; Some Problem Areas," *National Association of Secondary-School Principals Bulletin*, 43: 65–7, May 1959.

Discusses identification of gifted student, amount and nature of mathematics to be taught, special provisions for students, and traits of teachers of gifted students.

Payne, Joseph N., "Self-Instructive Enrichment Topics for Bright Pupils in High School Algebra," *The Mathematics Teacher*, 51: 113–17, February, 1958.

This study shows that bright pupils can do considerably more than they are presently required to do.

Peckman, Eugene F., "Providing a Challenging Program in Mathematics and Science for Pupils of Superior Mental Ability," SCHOOL SCIENCE AND MATHEMATICS, 52: 187–92, March 1952.

High Schools should—for selected pupils—set such an achievement as entering college on advanced standing as a definite goal and take every means to prepare these students for it.

Penk, G. L., "St. Paul Vitalizes Science and Mathematics for the Gifted," *American School Board Journal*, 138: 19–21, March 1959.

Shows what one school has done to meet needs for a mathematical and scientific curriculum geared to the space age. Outlines mathematics as well as scientific program.

Price, G. Baley, "A Mathematics Program for the Able," *The Mathematics Teacher*, *44*: 369–76, October, 1951.

A good program includes the gifted student, education to the limit of their abilities, counseling, and stimulation of interest by means of excellent teachers.

Reeve, W. D., "Problem of Varying Abilities Among Students in Mathematics," *Mathematics Teacher*, 49: 70–8, February 1956.

Gives answer as how to provide for varying abilities, how to provide a variety of courses for pupils of varying abilities and what activities can be used in conjunction with classroom work, Contains many references to current activities.

Reeve, W. D., "General Mathematics in the Secondary School," *Mathematics Teacher*, 47: 167–79, March 1954.

Actually outlines general mathematics plan for secondary schools through the junior college. Discusses in addition informal geometry and the insertion of geometry topics into ninth-grade work. Speaks of need for unification of algebra and trigonometry in the ninth grade also. Calls for inclusion of calculus in the twelfth grade. Author in favor of more meaningful mathematics for average student, but approach is equally applicable to the gifted student.

Richardson, M., *Fundamentals of Mathematics*. New York: The Macmillan Company, 1958. 507 pp.

An excellent book to be used as a text or supplementary text for a twelfth grade class of gifted students.

Roudebush, E., "Seattle Project for Talented Students," *National Association of Secondary-School Principals Bulletin*, 43: 78–81, May 1959.

Outlines program started in 1958. Defines selection of gifted students and their teachers. Discusses flexibility of program.

Strang, Ruth, chairman, "The Gifted Child," *Journal of Teacher Education*, 53: 210–32, September 1954.

Presents symposium on gifted child of which "Teachers for the Gifted," pp. 221–224, is of particular interest.

Scott, Laura D., Dona Small and Wm. W. Matson, "Mathematics Classes for Exceptionally Endowed Students in the High Schools of Portland, Oregon," July, 1956. 100 pp.

This publication describes the school program for the talented in Mathematics in Portland, Oregon and some of the pitfalls which were encountered.

Stephens, Harold W., "A Mathematics Club for Future Mathematicians," School Science and Mathematics, 54: 715–18, December, 1954.

This article is a report of the activities of a Mathematics club for future mathematicians and research scientists.

Stochl, James E., "Modern Mathematics in a Summer High School Course," *The Mathematics Teacher*, 52: 40–41, January, 1959.

One way of introducing high school students to modern mathematics is to offer a summer school course.

"The Mathematics Program at the Philips Exeter Academy," *The American Mathematical Monthly*, 58: 705–7, November, 1958.

A brief description of the type of course being offered at Phillips Exeter Academy. The editor states that this curriculum could be used in most college preparatory mathematics groups.

Vance, E. P., Editor, *Program Provisions for the Mathematically Gifted Student in the Secondary School.* National Council of Teachers of Mathematics, 1957.

This report has been directed to the task of calling the attention of secondary mathematics teachers to the fact that an increasing amount of attention is being devoted to the education of those students who are gifted in mathematics.

Wells, David W., "Modified Curriculum for Capable Students," *Mathematics Teacher*, 51: 181–83, March 1958.

Presents results of first-year algebra in eighth grade. Students then move on to take second-year algebra in the ninth grade. Outlines mathematics program through twelfth grade for these students. Indicates use of student aides to assist teacher in individual study.

Wiesenthal, I. Theodore, "Science and the Intellectually Gifted Pupil," *High Points*, 41: 61–63, November 1959.

Described the two processes essential to effective science teaching: analysis and synthesis. Gives method for stimulating creativity in science. Some presentation useful in mathematics.

Wilson, Jack D., "Trends in High School Mathematics," School Science and Mathematics, 55: 462–68, June, 1955.

A report on identification of trends in mathematics. The need of the nontraditional mathematics is made more vivid.

Wirszup, I., "Some Remarks on Enrichment," *Mathematics Teacher*, 49: 519–27, November 1958.

Takes up aspects of enrichment to broaden and enliven a student's experience in high school mathematics. Cites enrichment practices in east Europe and compares them with those of the United States. Ends with practical suggestions for enrichment in our high schools. Points out enrichment should be for both the student and teacher.

Wolfle, Dael, "Future Supply of Science and Mathematics Students," *Mathematics Teacher*, 46: 227–29, May 1953.

Tells of need for developing science and mathematics students to meet demands of industry and research. Indicates need of analysis of school and college populations to determine quality and quantity of replacements available. Also points up task of identifying high school students with mathematics and science potential. Suggests various scholarships to enable potential scientist to go to college when ordinarily he couldn't afford to.

II. Material Helpful to the Student

Bakst, Aaron, *Mathematics, Its Magic and Mastery*. New Jersey: D. Van Nostrand Company, Inc., 1952. 704 pp.

This book presents many interesting every-day problems to the reader. The author tries to show that mathematics is no more difficult than our everyday thinking. It is an application of common sense to situations that confront us.

Bell, Eric T., *Men of Mathematics*. New York: Simon and Shuster, 1937. 579 pp.

A collection of biographies of the great men of mathematics. Vivid accounts that bring the lives of these men into the view of the senior high school reader are given in detail.

Berkeley, Edmund Callis, and Lawrence Wainwright, *Computers, Their Operation and Applications*. New York: Reinhold Publishing Corporation, 1956, 331 pp.

Newest developments in both the techniques and equipment of automatic computing are described. Here we find practical information on how these machines work and what they can do.

Clair, H. S., "The Laws of Algebra and Modern Algebras," School Science and Mathematics, 53: 29–33, January 1953.

The article describes several types of number systems in which one or more of the commonly accepted laws of operations do not hold.

Courant, Richard, and Herbert Robbins, *What is Mathematics?* New York: Oxford University Press, 1941. 486 pp.

This book gives a formal presentation of much of the modern concepts of mathematics. It deals with material that ought to be in the high school curriculum, especially for the mathematically talented.

Cundy, H. M., and A. P. Rollett, *Mathematical Models*. London: Oxford University Press, 1952. 234 pp.

This book is designed for the student of mathematics who has a desire to show that mathematics can be fun and still be useful and meaningful. This book can be used by the student who is also gifted in the field of construction.

Friend, J. Newton, *Numbers, Fun and Facts*. New York: Charles Scribner's Sons, 1954. 191 pp.

This book shows how interesting, indeed fascinating, is the study of numbers, their origin, and peculiarities.

Gamow, George, *One Two Three . . . Infinity*. New York: The Viking Press, 1957. 335 pp.

This book is written as an attempt to collect the most interesting facts and theoreis of modern science in such a way as to present a general picture of the universe in its microscopic and macroscopic manifestations.

Heinke, Clarence H., "Variation—a Process of Discovery in Geometry," *The Mathematics Teacher*, 50: 146–54, February 1957.

Variation as a process whereby interesting geometric theorems are suggested is explored and its advantages noted.

Kline, Morris, *Mathematics in Western Culture*. New York: Oxford University Press, 1953. 472 pp.

This book answers questions pertaining to contributions which mathematics has made to western life and thought aside from techniques that serve the engineer.

Logsdon, Mayme I., *A Mathematician Explains*. Chicago: University of Chicago Press, 1935. 181 pp.

This book is written for those who have reached adult years with a distaste in mathematics. However, there are good chapters, for example, the chapter on logically building the real number system. These chapters could be used for the student's benefit.

May, Kenneth O., *Elementary Analysis*. New York: John Wiley and Sons, 1952. 597 pp.

This book could be used as a supplementary text for the gifted child, especially in a twelfth grade mathematics course. It is essentially directed to those who are going to take more advanced courses in mathematics.

Meyer, Jerome S., *Fun With Mathematics*. New York: The World Publishing Company, 1952. 176 pp.

This book is a recreational mathematics book containing mathematical gems that will delight and astound all those who are fascinated by the magic of numbers.

Ogilvy, C. Stanley, *Through the Mathescope*. New York: Oxford University Press, 1956.

A popular discussion of selected topics concerning number theory, algebra, geometry, and analysis.

Rademacher, Hans, and Otto Toeplitz, *The Enjoyment of Mathematics*. New Jersey: Princeton University Press, 1957. 197 pp.

This book introduces the reader to some of the fundamental ideas of mathematics; the ideas that make mathematics exciting and interesting.

Ransom, William R., *One Hundred Curious Mathematical Problems*. J. Weston Walch, 1955.

A group of problems which are difficult but which a curious mathematically inclined student would enjoy to work is the basis for fascinating reading.

Rees, Mina, "Modern Mathematics and the Gifted Student," *The Mathematics Teacher*, 46: 401–6, October, 1953.

This article is concerned with the world that confronts the gifted student of mathematics upon graduation. A brief resume of the current activity in mathematics that might challenge the interest of gifted high school students is discussed.

Rein, Constance, *From Zero to Infinity*. New York: Thomas Y. Crowell Company, 1955. 145 pp.

This book undoubtedly would motivate many to more serious thinking in number theory. It serves well to get others interested in the fascinating theory of numbers.

Williams, J. D., *The Compleat Strategyst*. New York: McGraw-Hill Book Company, Inc., 1954. 217 pp.

A recreational mathematics book which deals with the theory of games of strategy. Compare this with games of chance.

MANTLE OF EARTH LIKE STEEL, PLASTIC MATERIAL

Geophysical studies indicate that the mantle of the earth is as rigid as steel during short periods of time, but is more like a plastic material over a multi-millon year span.

Earthquake waves, which may last a few minutes, have shown that the earth, like a steel ball, can be deformed by strong forces, but springs back to its original shape after the forces are removed.

On the other hand, geological processes indicate that over millions of years the earth can be permanently deformed, like a plastic, as when mountains rise where seas once existed.

Chemistry in the Secondary Schools of America
A Historical Treatment*

Gerald Osborn
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Through a study of the history of education in the United States, we learn that secondary school science in the United States was first fostered by the academies and that the first academy was founded in Philadelphia in 1751. During the last quarter of the 18th century, academies were offering courses in astronomy, natural philosophy, botany, and agriculture. Early in the 19th century, these same academies began offering chemistry. By 1825, a total of twelve academies were offering a rather bookish course in chemistry. Demonstrations were performed by the teacher, but the student got no opportunity to do individual work in a laboratory. One can probably assume that modern educators would not approve of the methods used in those early chemistry courses. We are further told that most of the scientific apparatus, chemicals, and textbooks were imported from England.

The first high school established in the United States was the Boys' High School of Boston which was opened in 1821. Just five years later a Girls' High School was also organized in Boston. These first high schools offered chemistry as an elective subject, and the course was patterned after that of the academy—a book subject with little or no laboratory work beyond the lecture demonstrations given by the teacher. Facts, principles, laws, and theories were learned by rote. All of this was complicated by the fact that certain forces opposed the teaching of chemistry in high school. For example, religious people called chemistry the "black-art" and considered it to be "the work of the devil." Another factor in opposition was the economic one, namely, the high cost of equipment and chemicals.

By 1850, high school chemistry was considered to be a worthwhile practical subject. The Michigan State Normal College (now known as Eastern Michigan University) was the first teacher's training school west of the Alleghany Mountains; it was founded at this time and the law** which provided for the creation of this college specified

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* Editor's Note: In recent years, the journal has published two excellent articles on the history of biology[1] and general science.[2] Our Chemistry Departmental Editor is an authority on the history of Chemical Education. Hence, at the request of the Editor, he has prepared this article as a follow-up to our earlier ones.

[1] Rosen, Sidney, "The Origins of High School General Biology." School Science and Mathematics, LIX (June 1959) 473–89.

[2] Webb, Hanor A., "How General Science Began." School Science and Mathematics, LIX (June 1959), 421–30.

** Michigan Act 139 P. A. 1850, Section 2

"That a State Normal School be established and confined at Ypsilanti in the County of Washtenaw upon the site selected by said board of education, the exclusive purposes of which shall be the instruction of persons, both male and female, in the art of teaching and in all the various branches that pertain to a good common school education. Also to give instruction in the mechanic arts of husbandry and agricultural chemistry; in the fundamental laws of the United States, and in what regards the rights and duties of citizens."

that instruction was to be given in agricultural chemistry along with other practical subjects listed.

The idea of laboratory instruction, as we have seen, was imported from Germany. It was during the second quarter of the 19th century that a famous German chemist, Justus Liebig, in his laboratory at Giessen began to teach his students by the assignment of laboratory space and individual experiments. The practice gradually attracted attention and caused the subject of chemistry with individual laboratory work to become a part of the offerings of German secondary instruction. Furthermore, science was introduced into the elementary education by the spread of Pestalozzianism in Prussia and the other German States. Before the middle of the 19th century, elementary science was a part of the "Volkschulen."

In order to understand the status of chemistry instruction in the colleges of the United States around the middle of the 19th century, F. H. Getman in his "Life of Ira Remsen" tells the following story which was at one time reported by Dr. Remsen. "In those days," said Remsen, "the medical student was assigned to a preceptor, and my preceptor was the professor of chemistry. Under his guidance, I had learned much chemistry by reading out of a book. . . . On one occasion while reading from a textbook of chemistry, I came upon the statement 'nitric acid acts upon copper.' I was getting tired of reading such absurd stuff, and I was determined to see what this meant. Copper was more or less familiar to me, for copper cents were then in use. I had seen a bottle marked 'nitric acid' on a table in the doctor's office where I was then doing time. I did not know its peculiarities, but I was getting on and likely to learn. The spirit of adventure was upon me. Having nitric acid and copper, I had only to learn what the words 'act upon' meant. Then the statement, 'nitric acid acts upon copper' would be something more than mere words. In the interest of knowledge, I was even willing to sacrifice one of the few copper cents then in my possession. I put one of them on the table, opened the bottle marked 'nitric acid' poured some of the liquid on the copper and prepared to make an observation. But what was this wonderful thing which I beheld? The cent was already changed, and it was no small change either. A greenish blue liquid foamed and fumed over the cent and over the table. The air in the neighborhood of the experiment became colored dark red. A great colored cloud arose. This was disagreeable and suffocating—how could I stop this? I tried to get rid of the objectionable mess by picking the coin up and throwing it out of the window, which I had meanwhile opened. I learned another fact—nitric acid not only acts upon copper but it acts upon fingers. The pain led to another unpremeditated experiment. I drew my fingers across my trousers and another

fact was discovered. Nitric acid acts upon trousers. Taking everything into consideration that was the most impressive experiment, and, relatively, probably the most costly experiment I ever performed. I tell of it even now with interest. It was a revelation to me. It resulted in a desire on my part to learn more about that remarkable kind of action. Plainly the only way to learn about it was to see its results, to experiment, to work in a laboratory." It was such experiences as this one which led Ira Remsen in 1867 to abandon his plan to study medicine and instead to pursue the study of chemistry. Since at that time the American colleges had little to offer in the way of laboratories or as advanced studies in chemistry, Remsen decided to go to Munich and study under Justus Liebig, the real pioneer in laboratory instruction. After completing this education abroad, Remsen returned to America and became the first to occupy the chair of chemistry at the newly organized Johns Hopkins University where he eventually became president of the university. It was this same Ira Remsen who worked out an effective method for the synthesis of saccharin.

Many leading scientists, such as Thomas Huxley in England and President C. W. Eliot of Harvard, stressed the value of sciences for complete living and social progress. Eliot accomplished a service for the sciences largely by an extension of the elective system and by placing an emphasis upon science in the curriculum of school and college.

Following the Civil War, chemistry grew rapidly as a subject of practical value. Since many of our chemists had been educated in Germany they brought back to our colleges and universities the concept of the individual laboratory instruction. Sometime in the last quarter of the 19th century, high schools offering chemistry provided in a more or less limited fashion for laboratory work.

It was not until around 1870 that the colleges and universities began to accept chemistry for entrance requirements and soon thereafter they set up definite requirements for high school chemistry in accredited schools. This led to a period which was referred to as "college domination." The high school course was merely a "cut down" version of the freshman college course and the high school texts were reduced versions of the college texts. In many cases the explanatory and descriptive material were either omitted or given in fine print. This can be shown by going back to around 1920 and making a comparison of Alexander Smith's high school text to that of his college text. Here it will be seen that the topics included are the same and the order is also almost identical to that of the college text.

Around the turn of the century, three reports appeared, that of the Committee of Ten in 1893, that of the Committee on College En-

trance Requirements in 1896, and that of the Carnegie Foundation for the advancement of teaching; all tended to carefully outline the separate disciplines in terms of what was needed for college entrance. Thus college preparation became the main objective for the study of secondary science. Since the high school attendance increased notably from 1900 to 1910, the National Education Association (1913) appointed a commission on the Reorganization of Secondary Education. This commission produced two reports: the first, "Cardinal Principles of Secondary Education" appeared in 1918; the second, entitled "Reorganization of Science," appeared in 1920. These two reports emphasized the functional value of science. The "Thirty-first Year Book of National Society for the Study of Education" stressed the impact of science on human affairs and indicated that science courses should be organized around scientific principles and important generalizations. The greatest contribution was the plan to organize subject matter and learning experience around interpretive generalizations which are significant for a liberal education rather than around the laws and theories of pure science. These latter reports tended to free the high schools from "university domination." High school teachers began to plan their own science curricula and also outlined their own courses in terms of community needs. High school teachers of chemistry rather than university professors wrote the textbooks. As a result, many practical every day topics, such as chemistry in the home, chemistry and health, and soil chemistry were included in the courses.

In spite of the new-won freedom, high school teachers still pattern their courses after those taught in the universities. The author recently wrote an article (an unpublished article which will appear in an early issue of "The Detroit Metropolitan Science Review") on the trend of college chemistry teaching during the past 50 years in which he showed that less emphasis is now placed on descriptive chemistry and more emphasis is placed on the study of scientific laws and basic principles. There is an increasing tendency to explain chemical and physical properties on the basis of the structure of matter. A comparison of the high school textbook of 1910 to that of 1960 shows exactly the same trend. In this connection, within the past ten years, college general chemistry texts have emphasized the sub-orbital arrangement of electrons and a few of the high school texts are now doing likewise. Another example should also be mentioned. At Brown University and other colleges there has been an effort to organize general college chemistry around the covalent bond idea. The main purpose of this innovation was to emphasize organic chemistry during the freshman year, and thus one would not be duplicating a high school course. However, recently at Reed College a group of high

school teachers, aided by college teachers, are now developing a high school course along this identical plan.

The recent publication of the National Society for the Study of Education, Part I "Rethinking Science Education," in referring to the work of the Physical Science Study Committee (working out of Massachusetts Institute of Technology) stated that this committee is "aiming at the reduction of the number of topics conventionally covered in these courses, and is concentrating upon the development of a better understanding of a few conceptual patterns which are fundamental in modern thinking regarding these fields." The committee further states "the laboratory work for these new courses is more experimental and more directly orientated to problem solving than the conventional cook book type.

The author of this paper would like to see the high school chemistry course kept somewhat descriptive with emphasis on laboratory work. If high school students are given a good background in mathematics and high school physics along with a descriptive course in chemistry that stresses elementary quantitative relationships they will have an adequate background for the pursuit of college chemistry.

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NATIONAL GEOGRAPHIC SOCIETY LAUNCHES 39TH YEAR OF WEEKLY SCHOOL BULLETINS

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Rational Numbers and Terminating Placemal Fractions

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It is a popular pastime of the day to find properties of numbers which are not properties of the numbers at all, only of their numerical representatives. One of the more obscure of these is that of the property of forming unending decimal numbers. It is true enough that rational numbers which have unending decimal forms exist, but the unendedness is a property of the decimal number system, not of the rational numbers. On the other hand, irrational numbers always give unending fractional forms no matter what integer is used as the base of the number system.

Proof of the first contention is merely one of exhibiting the existence of a terminating placemal form for any rational number. Confining the balance of this discussion to rational numbers less than one (the number of digits to the *left* of the point is immaterial), one can change a rational number of the general form p/q to a simple placemal form by assuming a number system with base q. As p is less than q (p/q is less than one), p is represented by a single digit. This, then, is the transformation: $p/q = \cdot p$ (to base $q\cdot$). There are other bases which could be used (multiples of q), but one is enough to demonstrate the contention.

The second contention to be supported is that an irrational number can never be represented by a periodic or terminating placemal form. On page 67 of *What Is Mathematics?*, Courant and Robbins demonstrate that all periodic decimals are rational numbers. They also remark that the proof in the general case is essentially the same. Thus, we have the theorem that, no matter what the base, any periodic or terminating placemal fraction is a rational number. Thus, if one assumed an irrational number to have such a form, it could be shown that this form would lead to a rational number, which would be a contradiction. This leaves as the only possible form for irrational numbers in any placemal system, the non-terminating, non-periodic one.

These considerations serve to simplify one's notions of the differences between rational and irrational numbers expressed as placemal numbers: A rational number may be expressed as a terminating placemal number by proper choice of base; an irrational number may be expressed only as a non-terminating placemal number, no matter what the base.

Linear Indeterminate Equations—An Aid to Enrichment

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Each year the instructor of intermediate algebra spends much time on systems of linear equations, stresses the importance of having an equal number of equations and unknowns in order to determine a unique solution for the system and virtually, if not completely, ignores the parametric solutions of a system with more unknowns than equations. In the general solution of the equations, if there are n equations in $(n+a)$ unknowns, n unknowns may be be expressed in terms of the other a unknowns, which are arbitrary. If the solutions of the system are restricted to integers (or rational numbers) only, all the unknowns may be expressed in terms of a arbitrary parameters. If further restrictions are imposed it is often possible to obtain a unique solution. The simplest type of linear indeterminate equation is a single equation in two unknowns. These may be used to liven up many a class discussion if they are artfully contrived and can create interest on the part of the better students. Puzzles which take on an algebraic form for solution are a part of the folk-lore of many nations and can be created from whole cloth as well. Consider the following problem:

Five buses, each containing an equal number of persons and nearly filled to their capacity of 54 each, deposit their passengers at a train station where seven persons are already waiting for the train. When the train arrives, the passengers are divided equally among fourteen cars. How many boarded the train?

This problem reduces to the solution of the equation

$$5x+7=14y \tag{1}$$

where x represents the number on each bus, y the number boarding each car and each side of the equation the total number of people. Now any algebra student knows that equation (1) is the equation of a line and that the equation of a line has every point (x, y) on it as a solution. The nature of the problem, however, limits the solutions to positive integers only and specifically asks for the largest value of x less than 54.

The method of solving such linear indeterminate equations can be illustrated by means of a somewhat simpler example such as the following problem:

There exist two positive integers such that the sum of one of them and three times the other is 21. Find the integers.

This gives us the equation

$$\begin{aligned} 3x+y&=21 \\ y&=21-3x \end{aligned} \tag{2}$$

For any integral value of x, y will be integral. The problem specifically asks for positive integers, however, so it is necessary that $x<7$. The solutions then, are as follows:

$$x= 1,\ 2,\ 3, 4, 5, 6$$

$$y=18, 15, 12, 9, 6, 3.$$

The solution could have been made unique by requiring that $x>y$.

Returning to equation (1); solving for x, we find

$$5x=14y-7$$

$$x=\frac{14y-7}{5}$$

$$x=2y-1+\frac{4y-2}{5} \tag{3}$$

In this case, in order for x to be an integer, it is necessary that

$$\frac{4y-2}{5}$$

be an integer. Let

$$n=\frac{4y-2}{5}\,.$$

Solving for y, we obtain

$$4y=5n+2$$

$$y=\frac{5n+2}{4}$$

$$y=n+\frac{n+2}{4}\,. \tag{4}$$

Since y and n are both integers, it is necessary that

$$\frac{n+2}{4}$$

be an integer. Let

$$p=\frac{n+2}{4}\,.$$

Solving for n, we obtain

$$n=4p-2 \tag{5}$$

where p is any integer. Substituting in (4) we obtain

$$\begin{aligned} y&=4p-2+p \\ y&=5p-2. \end{aligned} \tag{6}$$

Substituting in (3) we obtain

$$\begin{aligned} x&=10p-4-1+4p-2 \\ x&=14p-7. \end{aligned} \tag{7}$$

The equations (6) and (7) represent the general solution of equation (1) in terms of an arbitrary parameter p which, if an integer, will yield integral solutions. Since x and y are to be positive integers, it follows that p must be positive. Solutions to the equation (1) may be obtained by choosing any value(s) of $p=1, 2, 3, 4, \cdots$. The terms of the original problem, however, specified that x was nearly 54. Therefore we need the largest value of p such that

$$14p-7<54$$

$$14p<61$$

$$p<4\frac{5}{14}\cdot$$

The largest value of p which fulfills this condition is $p=4$, from which $x=49$, $y=18$, and the total number of passengers which boarded the train is 252.

A discussion of a few equations of this type may lead to the question of two equations in three unknowns. Such a system could arise from a problem such as the following:

A corporation wished to purchase a fleet of 100 automobiles for exactly \$250,000. They wished to buy automobiles of type A, costing \$2600 each, type B, costing \$2100 each and type C, costing \$1800 each. How many of each could they purchase?

This problem leads to the two equations

$$x+y+z=100 \tag{8}$$

$$26x+21y+18z=2500. \tag{9}$$

Eliminating z in equations (8) and (9) we obtain

$$8x+3y=700 \tag{10}$$

$$3y=700-8x$$

$$y=233-2x+\frac{1-2x}{3} \tag{11}$$

$$\frac{1-2x}{3}=n$$

$$x=-n+\frac{1-n}{2} \tag{12}$$

$$n=1-2p. \tag{13}$$

Substituting the result of (13) in (12), (12) in (11), and (11) and (12) in (8), we obtain the results

$$x=3p-1$$
$$y=236-8p$$
$$z=5p-135.$$

Since any positive value of p will result in a positive x, we solve the inequalities

$$8p<236, \qquad 5p>135$$

to obtain $27<p<30$. Letting $p=28$, 29, we obtain

$$x=83, 86$$
$$y=12, 4$$
$$z=5, 10.$$

The solution could have been made unique by requiring that they purchase more type B than C, or vice versa.

Problems such as the above are very easy to make up by observing that for any integers, p, q, there exist integers x, y, such that

$$px+qy=1$$

Often no positive integral solutions exist for a problem in terms of the stated conditions. In the automobile problem above, for example, if the cost of the fleet is to be \$200,000, the general solution is

$$x=3p$$
$$y=400-8p$$
$$z=5p-300.$$

In order for x, y, z, to be positive, it is required that $60<p<50$. No solution exists to the problem as stated then.

The use of such problems, which are an offshoot of number theory, requires a certain minimum background, usually obtained during the

course in intermediate algebra, but are admirably suited to the stimulation of interest in high school students and the improvement of the basic skills.

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SOIL TREATMENT MAY REDUCE STRONTIUM-90

The rate of uptake by soils of radioactive strontium-90 varies according to their composition and may be reduced by chemical treatment, new studies have shown.

Strontium-90 is a long-lived element of fallout from nuclear testing and in large doses is known to be a cause of leukemia and bone cancer in man. It acts like calcium; and well-developed calcium formations in both plants and animals tend to form a natural barrier to strontium-90.

Strontium-90 studies were made on crops grown on uncultivated but watered acidic, calcareous and alkaline soils.

The uptake from the calcareous soil was one-half to one-third the uptake from acidic soil. The investigators attributed this to the larger quantities of available calcium naturally present in calcareous soil.

Scientists found that it was possible to reduce uptake of radioactive strontium-90 by as much as 80% by adding calcium-high lime or gypsum to acidic soils.

Basic calcareous and alkaline soils treated with massive doses of phosphate showed a 50% reduction in uptake of strontium-90. Except for the phosphate treatment, however, none of the other procedures appreciably lowered strontium-90 uptake from calcareous soil.

The lack of cultivation while thorougly wetting the soil and allowing it to dry established to some extent the potential reduction of radioactive contamination through precipitation reactions. Further studies are needed to determine whether an appreciable part of this potential can be realized.

The availability of strontium-90 from soil to plants to man is an important factor in evaluating the health hazard presented by this radioactive element.

BACTERIA ABOARD ROCKET SHIPS COULD SURVIVE MOON TEMPERATURES

Earth-varieties of bacteria are rough and tough enough to survive extreme temperatures on the moon, two researchers have reported. Thus man-made rocket ships might carry bacteria to the moon where they will thrive.

They found that many types of bacteria can survive heat as intense as that presumed to exist in sunlight on the surface of the moon. Many scientists believe this temperature to be well above the boiling point of water on earth.

The two men found that many bacteria survived in a vacuum when the heat reached 275 degrees Fahrenheit, 63 degrees above the normal boiling point of water at sea level.

Many bacteria should be able to survive in the dry state of a vacuum on the moon's surface, the scientists believe.

A Modern Course in Chemical Qualitative Analysis

Charles H. Heimler
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College teachers of analytical chemistry, recognizing the shift in modern laboratory procedures from the use of traditional chemical methods to the use of techniques employing electronic instrumentation, have begun to direct the emphasis and approach in chemical education towards a more fundamental study of the basic principles and theories of chemistry.

The author, a teacher of chemistry at Orange County Community College, has followed this pattern, and has developed a sophomore course in chemical qualitative analysis that places emphasis on the study of basic concepts and principles, and their relationship to the behavior of matter. Further, the course has been designed so as to cultivate the problem-solving abilities of the students, and also, to encourage the student to enter into a modified form of independent laboratory experimentation. This shift in emphasis and organization in qualitative analysis has been tried by the author for several semesters and has proved to be very successful.

The course in qualitative analysis at Orange County Community College is based on a study of atomic structure, chemical bonding, and chemical equilibrium. The purpose of the laboratory work is to provide the student with practical examples and illustrations of the relationship between these principles and the actual behavior of chemical substances. A basic textbook[1] is used and, in addition, the bookshelf in the laboratory contains many additional texts and reference books. The students are also asked to buy their own copies of the *Handbook of Chemistry and Physics*.[2]

Three hours of lecture and recitation work are given, and each student is required to spend a minimum of five hours per week in laboratory work. (Additional laboratory time is provided and is desired by most students.) A strong attempt is made to correlate the theory and principles discussed in the lecture and recitations with the actual laboratory work. The author teaches both lecture recitations and laboratory sections, and firmly believes that this is essential for this type of course.

The introductory laboratory work is traditional in nature. A student is assigned a set of equipment and laboratory work space,

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[1] T. R. Hogness and Warren C. Johnson, *Qualitative Analysis and Chemical Equilibrium*. (New York: Henry Holt and Company, 1958).

[2] C. D. Hogman, R. C. Weast, and S. M. Selby, *Handbook of Chemistry and Physics, 40th Edition* (Cleveland: Chemical Rubber Publishing Co., 1958).

and then begins to carry out the laboratory directions as provided in the textbook. The student completes a set of preliminary experiments and is then given an oral quiz by the instructor. If the student passes the quiz, he is given an unknown chemical sample to analyze. However, if the student does not pass the test, he is asked to study some more and repeats the examination before he proceeds further. (A grade is given for both the quiz and the unknown report.) Each student is required to complete the analysis of a minimum of six unknown samples during the semester.

For about the first four weeks the instructor plays a dominant role in the laboratory work. Considerable discussion of laboratory techniques and interpretation of textbook laboratory directions is provided. However, after this introductory period the students are encouraged to proceed on their own. They are requested to depart from the traditional method of analysis, as provided in the class textbook, and to develop schemes of analysis based on the chemical principles and properties they have either studied in class, or can locate in the available reference books. The reference book *Spot Tests*[3] has proved particularly useful.

A significant part of the course work involves the completion of a student project. The project deals with a report on the relationship between qualitative analysis and some other science in which the student is interested, or it may deal with a specific laboratory technique in qualitative analysis. The project must involve preliminary library research, actual laboratory experimentation and the completion of a written report. For example, last semester a student, who was also taking course in geology, completed a project dealing with the identification of the trace elements in the mineral magnetite. Another student worked on the identification of the elements by spectroscopic means, and completed a report dealing with the use of the Fisher-Todd spectranal in the analysis of metallic samples.

The final examination in this course is of the open-book type. Students are allowed to bring their text, and any other reference books they wish. The examination contains both descriptive type questions and quantitative problems—the object being to evaluate the student's ability to use his knowledge and understanding of chemical principles in the solution of practical analytical problems.

The results of this type of course have been very favorable. The stimulation of interest on the part of the students and the kindling of their enthusiasm has been very impressive. Although they were required to spend a minimum of five hours in the laboratory, most students were willingly working from eight to fourteen hours per week, and sought additional time during their vacation periods. In

[3] Fritz Feigl, *Spot Tests*, Volume I, *Inorganic Applications*. (New York: Elsevier Publishing Co., 1954).

addition, about one-half of the class turned in project reports of exceptional quality. Moreover, the students developed a deeper insight into the nature of analytical procedures and acquired a greater sense of maturity about laboratory work. Further, their achievement on standardized tests (American Chemical Society Examinations) indicated that their knowledge of factual material was equivalent to that possessed by students taught by the instructor in a more traditional manner.

An Eye for an *i*

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Let us cast an eye on i for a moment. There are two numbers used in algebra whose squares are minus one. Twins are often hard to tell apart: these two, i and j, are easy to tell apart, for one is the negative of the other. But there is no way to tell *which is which!* The equation $x^2+1=0$ shows that $i+j=0$, and $ij=1$. At first you think it is easy, because j is the cube of i: but just the same i is the cube of j, and each is the reciprocal of the other.

We do not need them both, for either is simply expressed in terms of the other. Electricians prefer to use j, mathematicians commonly use i. What are they good for? Three conspicuously different uses may be cited.

The most extraordinary use of them in Euler's formula

$$e^{jx}=\cos\ (x \text{ radians})+j\sin\ (x \text{ radians})$$

which enables physicists to treat simple harmonic motion by means of exponential formulas.

Another surprising use is in finding the equation of a circle through three given points, say (a,b), (α,β), and a third point. It is easy to see that the first two points satisfy the equation

$$k[(x-a)(b-\beta)-(y-b)(a-\alpha)]=(x-jy-a+jb)(x+jy-\alpha-j\beta)=0$$

and that this contains x^2+y^2 and no xy term, and so is the equation of a circle. Then we need only determine k so that the third point satisfies it also. If you like algebraic fun, try this for a circle through (0, 0), (0, 1), and (1, 0). You should get $k=j-1$.

Thirdly, there are equations where the terms in j cannot cancel with the other terms, and one such case shows how to get the square root of $a+bj$ in the form $x+yj$. For

$$a+bj=(x+yj)^2=x^2-y^2+2xyj$$

leads to

$$a = x^2 - y^2$$
$$b = 2xy$$

whence

$$(x^2 + y^2)^2 = a^2 + b^2 = R^2 > a^2.$$

Then from

$$x^2 + y^2 = R$$
$$x^2 - y^2 = a$$

we get

$$x^2 = (R + a)/2$$
$$y^2 = (R - a)/2.$$

and since $R > a$, both x and y come out real.

But what we started to say was that the "-1" ought to come out from under the radical sign. We used to hear about the Professor who had a nightmare in which he found himself with a negative sign under a radical, and could not get out. There is no need to admit a negative number under a radical sign: factor out the i. We get no benefit from the form $\sqrt{-1}$: i or j is better. That $i^2 = -1$ is the useful and all sufficient thing to say about i. Let us drop the cumbersome, inutile, and provocative form $\sqrt{-1}$.

Also, it would be better to carry on with j, (as the electrical people do), than with the symbol i, which suggests the unfortunate term "imaginary." Gauss, whose opinion is worth adopting, proposed to use the term "lateral" rather than imaginary. The complex number $a + bj$ is real if $b = 0$, and is lateral if $b \neq 0$.

STATE SCIENCE BULLETINS

The Sarasota County Association of Science Teachers publishes an interesting bulletin known as SCAST BULLETIN. It contains many news notes and teachings tips. It is edited by Dempsey L. Thomas, Riverview High School, 4950 Lords Lane, Sarasota, Florida.

The Michigan Science Teachers Association also publishes a bulletin, called THE MSTA NEWSLETTER. Each year the Association adopts one central theme around which its year's activities and annual convention are organized. One issue of the *Newsletter* contains an extensive report of that year's activities. Copies of the five annual reports are free on request. Write:

Michigan Science Teachers Assocation
535 Kendall Avenue
Kalamazoo, Michigan

Thermal Conductivity of Gases

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A simplified method for determining the approximate relative thermal conductivity of gases has been described in a previous study.[1] In that study it was reported that the relative thermal conductivity of a gas may be determined by measuring its rate of cooling in a suitable cell of known volume and surface area. The equation $k=C_v nr/A$ was used in calculating conductivity values in the previous study. This method was found suitable for student determinations or in general for obtaining good approximate values in a greatly simplified manner.

Subsequent investigation revealed that a linear relationship exists between k/C_v and r_g/r_a where k is thermal conductivity, C_v is molar heat capacity of the gas at constant volume, r_g is rate of cooling of the gas in degrees per second, and r_a is rate of cooling of air in degrees per second.

Relationship of k/C_v and r_g/r_a

When k/C_v is plotted against r_g/r_a for the eleven gases used in the study a straight line is obtained with a slope of 1.098×10^{-5} (see Table I and Figure 1). This relationship may be expressed by the equation

(1) $$k/C_v = mr_g/r_a.$$

Rearrangement of (1) gives

(2) $$k = mr_gC_v/r_a$$

in which m is the slope of the line obtained in figure 1.

Evaluation of m

Graphical Evaluation of m:

In figure 1 two points P_1 and P_2 were selected at random along the line obtained in the figure. The value of the slope, m, was then computed by use of the formula $m=Y_2-Y_1/X_2-X_1$ and found to be 1.098×10^{-5}, average value.

Mathematical Evaluation of m:

Evaluation of m from accepted values of k and C_v taken from the Handbook of Chemistry by Lange is described in detail as follows:

Previous investigation indicated that the formula

[1] Herald, Aurlocolus C., "A Simplified Method for Determining the Approximate Relative Thermal Conductivity of Gases," School Science and Mathematics, January, 1960, pp. 10–14.

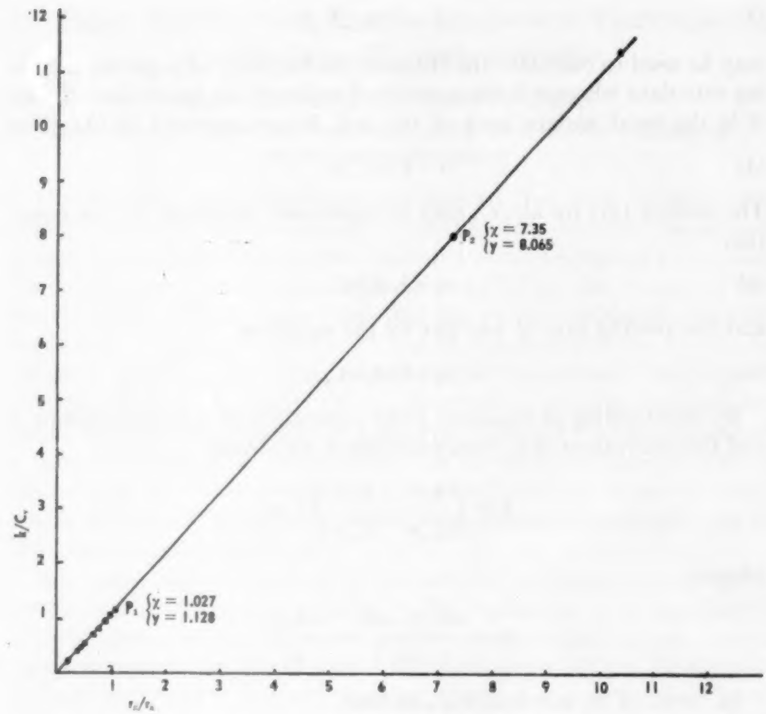

FIG. 1. k/C_v plotted against r_g/r_a. Plotted points taken from Table I.

TABLE I

Gas	r_g	r_g/r_a	C_v	$k\times10^5$	k/c_v
Cl_2	.0557	.271	6.14	1.83	.298
H_2S	.0865	.421	6.57	3.045	.463
CO_2	.0916	.446	6.92	3.39	.489
C_2H_2	.1039	.506	7.93	4.4	.554
NH_3	.1460	.711	6.57	5.135	.781
NO	.1522	.741	5.11	4.16	.814
CH_4	.2040	.993	6.59	7.2	1.092
Air	.2053	1.000	5.07	5.572	1.099
O_2	.2110	1.027	5.05	5.7	1.128
H_2	1.509	7.35	4.91	39.6	8.065
He	2.108	10.26	2.98	33.6	11.275

r_g = rate of cooling of gas in degrees per second
r_g/r_a = ratio of rate of cooling of gas to rate of cooling of air
C_v = molar heat capacity at constant volume
$k\times10^5$ = thermal conductivity of the gas
k/C_v = ratio of thermal conductivity to molar heat capacity.

$$k = C_v nr/A \tag{3}$$

may be used to calculate the thermal conductivity of a gas from cooling rate data where n is the number of moles of the gas in the cell and A is the total surface area of the cell. Rearrangement of (3) gives

$$r = kA/C_v n. \tag{4}$$

The cooling rate for air, r_a, may be expressed, therefore, by the equation

$$r_a = k_a A/nC_a \tag{5}$$

and the cooling rate of any gas by the equation

$$r_g = k_g A/nC_g. \tag{6}$$

By substituting in equation 2 the equivalent of r_a from equation 5 and the equivalent of r_g from equation 6 we obtain

$$k_g = \left(\frac{k_g A}{C_g n} \div \frac{k_a A}{C_a n}\right) C_g m$$

whence

$$k_g = \frac{mk_g A C_a n C_g}{C_g n k_a A} = \frac{mk_g C_a}{k_a}.$$

In terms of m, $m = k_g k_a / k_g C_a$ so that

$$m = k_a/C_a. \tag{7}$$

From Lange's Handbook of Chemistry $k_a = 5.57 \times 10^{-5}$ and $C_v = 5.05$. Substitution of these values in equation 7 and solution of the equation for m gave 1.098×10^{-5} as the value for m which is in agreement with the value obtained graphically (Figure 1).

Summary

It appears that the constant m is not only universal but independent of experimental conditions. The calculation of approximate relative thermal conductivity of any gas can therefore be further simplified by making use of equation 2 where $m = 1.098 \times 10^{-5}$. Use of this equation eliminates the necessity for determining the numerical value of the cell constant. One simply determines the ratio of the cooling rate of any gas to that of air in the same experimental cell and computes directly the thermal conductivity, k_g, of the gas.

Further investigation is being conducted relative to the linear relationship between k, time of cooling t, and molar heat capacity C_v.

Paper Negatives—A Simple, Inexpensive Technique for Obtaining Microphotographs

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Observations made at science fairs indicate that many students are interested in microphotography. Unfortunately though, the school budget doesn't allow for the purchase of a camera adapted to fit onto the microscope. Some students have attempted to take pictures without adapters. Others have tried making various types of adapters fitted with a close-up lens. Many times this leads to pictures which are out of focus or to a frustrating experience for the student. The purpose of this article is fourfold: (1) to illustrate a reliable method for obtaining photomicrographs of a variety of objects without the use of a camera; (2) to show by examples, the versatility of the procedure; (3) to bring out the simplicity of the method; and, (4) to present data which will enable the worker to make quantitative measurements of the object so photographed.

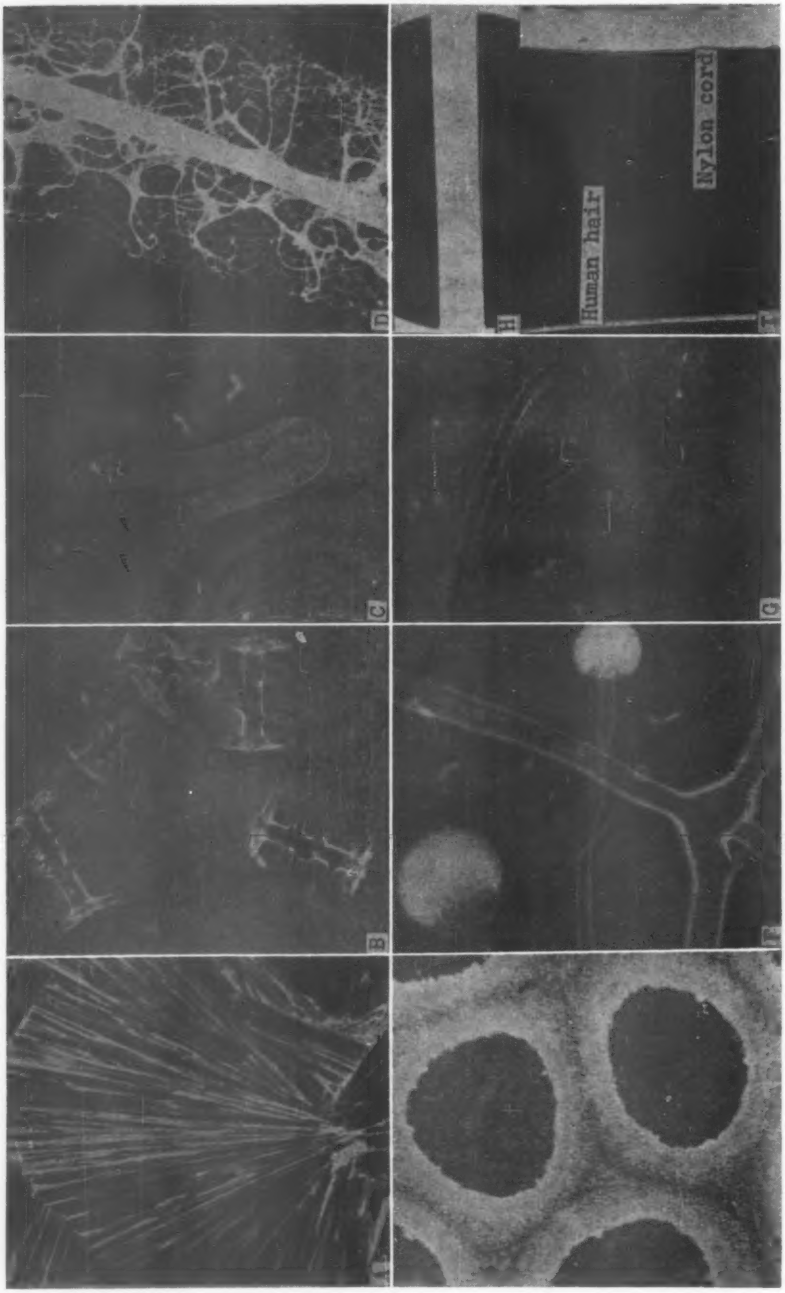
The paper negative method is not new. In fact many photographic books call it "negative printing." The technique is a specialized photographic process. It can be applied, for example, to any object small enough to be placed under the microscope. It can be equally utilized to record biological, chemical or physical specimens (Figure 1).

Photographic printing is a process of passing light through a negative so that it forms a corresponding image in the sensitive emulsion of a photographic paper. Since no camera is utilized in the procedure no negatives are obtained. The resulting prints are actually negatives. This feature does not distract from the usefulness of the method.

In choosing material to be photographed it should be remembered: those structures which were white or clear in the microscopic field will be dark on the developed paper and the dark areas or the blacks will be white in the finished product. The capillitium of the *Stemonitis* sp. (Figure 1D) is dark in reality. Exposed to photographic paper the capillitium is white and the background becomes dark. In Figure 1E the pore openings of the *Polyporus* sp. are clear on the microscopic slide and the tissue area is dark.

The microphotographs are made by the projection of the object through the microscope and onto photographic enlarging paper. The success of the technique is dependent on the precision of the optical system of the microscope and on the focusing ability of the operator.

Preliminaries to obtaining the microphotographs include darkening a work room, having the microscopic slides in order and cleaned, set-


Human hair
Nylon cord
D
H
C
G
B
A

ting up the monocular compound microscope, arranging the necessary trays, and preparing solutions. It is essential that preliminaries be taken care of so that once underway, the worker can proceed without interruption.

Selection of a dark room for exposing photographic paper need not be as critical as for the selection of a dark room for film development. Paper emulsions are relatively slow. Their sensitivity is limited only to blue light. A fair degree of illumination is permissible in the dark room so long as the light present does not contain blue.

Materials

A ring stand is placed on either side of the monocular compound microscope in the dark room. Burette clamps are then placed on the ring stand approximately 10 inches above the ocular lens of the microscope.

The following items can be purchased for less than $5.00 from any photographic store. Acid-fixer, stop bath, paper developer, 25 sheets of 4×5 medium contrast (f-2) enlarging paper, 3 plastic or enamel 5×7 trays, a safelight, and a 4×5 print frame.

The Eastman Kodak Company puts out a package called "Tri-Chem Pack." The package contains sufficient chemicals to make eight ounces of the necessary developer, stop bath and acid fixer. The cost is less than 40¢ for the package of three chemicals. There are enough chemicals for about two to three hours of processing.

It is essential that the paper purchased is enlarging paper and not contact paper. There are three basic types of photographic paper. For contact prints there are papers with slow emulsions. The enlarging papers have a fast emulsion. These two types of papers are known, respectively, as chloride and bromide papers. The chlorobromide paper is an in-between type which can be used for both contact and projection printing.

A very satisfactory low-priced safelight is obtainable at many photographic stores. The bulb screws into any drop cord or outlet. Of course there are safelights which are more expensive with a series of filters or others which are 2-way affairs.

The print frame holds the photographic enlarging paper. It is nec-

Legend for illustrations, left:

FIG. 1. Examples of the application of photographic paper technique to obtain negative prints of microscopic objects: A. crystals of mannitol; B. gemmule spicules of the fresh-water sponge *Meyenia subdivisa* Potts; C. nematode larva (order Rhabditida); D. capillitium of *Stemonitis* sp.; E. pore surface of a *Polyporus* sp.; F. sporangium of a Phycomycete species; G. gemmule and dermal spicules of the fresh-water sponge *Spongilla lacustris* (Linnaeus); H. paper clip; I. human hair and nylon cord.

essary that the emulsion side of the paper be facing the ocular of the microscope when the paper and the print frame are placed in the clamps. The emulsion is the glossy-surfaced side of the paper. As a rule the paper tends to curl toward the emulsion side. If desired, the enlarging paper can be held between two pieces of clear glass in place of the print frame. When the substage lamp is turned on for approximately 2 to 4 seconds, the image of the object strikes the paper. The paper is developed in the usual manner.

Focusing of the object can be done on ground glass supported on the ring stand. A substitute can be had by placing a 4×5 index card in the frame and focusing on the card. Onion skin paper supported by a thin piece of glass can also be used for focusing.

Procedure

Step 1

A. Place side by side the three plastic or enamel trays in the dark room. For convenience, the right hand tray is used for the developer, the center one for the stop bath and the tray on the left for the acid fixer.

B. Make some provision for washing the prints in running water after they are fixed. It will be necessary to wash the prints thoroughly in running water for approximately one hour.

Step 2

A. Place the microscope slide on the stage of the microscope.

B. With the condenser system and the diaphragm of the microscope critically focus and cut out any extraneous light.

C. Turn out all lights except the safelight and the substage lamp.

D. Focus the object onto the ground glass, index card or the onion skin paper.

Step 3

A. With all lights out except the safelight place a piece of enlarging paper in the print frame. Make certain that all other unused enlarging paper is safely covered.

B. Place the print frame in the burette clamps.

C. Expose the paper by turning on the substage lamp. Different types of microscopic subjects require different exposure times. It is best to make a test exposure sheet for each different field. The test sheet is made by exposing a section of paper for 1 second; another section for 2 seconds; the next for 3 seconds and so on. After developing one can then determine the test time to obtain maximum contrast.

D. Place the paper in the tray containing the developer. Follow the

directions on the packages purchased or consult a handbook on photography for the developing procedure. If after making the test print and developing it you find that it is gray, or lacks contrast you probably need a paper of different contrast.

Measurement of Microscopic Object

An object magnified by the microscope has attained its magnification in two steps. The first magnification results from the objective employed and the second by the eyepiece. The final magnification of the object is the product of the objective and the eyepiece. On the objective one will find a number with "×" behind it. This combination of symbols indicates the magnification of that particular lens. Most microscopes are supplied with 10×, 43×, and 97× objectives. The eyepiece is generally marked with 10× although others are interchangeable. Calculation of the total magnification is made by multiplying the objective magnification by the eyepiece magnification. For example, the objective magnification is 43× and the eyepiece magnification is 10×, the final magnification is the product of the two or 430. This indicates that the diameter of the object has been magnified 430 times.

Determination of the diameter of an object on photographic paper is complicated by the fact that for maximum definition the image formed on the paper is well past the eyepoint. The eyepoint is that place in the area above the ocular where the emerging light rays cross (Figure 2). If the eye is placed approximately one inch above the ocular it will receive the greatest number of rays from the microscope and thus see the largest field. If the eye is too far from, or too near the ocular, part of the rays cannot enter the pupil of the eye and the microscopic image is restricted.

Figure 2 illustrates the principle of the eyepoint phenomena in microscopy. The photographic paper functions the same as the retina of the eye in that the paper records whatever is exposed to it. As the conical field increases so does the enlargement of the microscopic field. The larger the angle, however, the less definition of the subject on the paper.

Relative size of the object photographed is determined in three steps. First, determine the total magnification of the microscope. Second, determine the distance between the ocular and the enlarging paper. Third, compare the size of the photographed image with the size of a reference.

Once the total magnification of the microscope is determined refer to Figure 3 to find the "enlarging factor." The product of the height multiplied by the enlarging factor will be the number of times that the object has been magnified. For example, using a 10× objective and

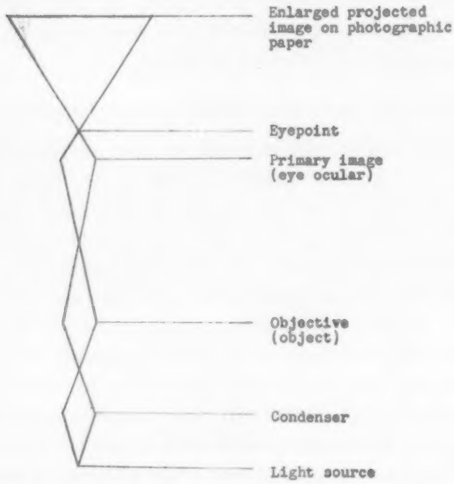


FIG. 2. The optical pathway of a compound microscope. The sketch illustrates the phenomena of the eyepoint and the enlargement of the object on photographic paper due to the increase in the conical angle beyond the eyepoint.

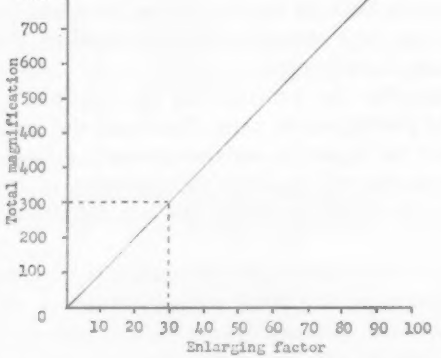


FIG. 3. Correlation diagram representing the parallel relationship between the total magnification of the microscope and the enlarging factor for the photographic paper.

an 8× ocular, the paper 5 inches from the ocular, then the enlarging factor is approximately 7.6. The product of the enlarging factor times the height is 38.0. The diameter of the object photographed has been magnified 38 times.

The reference used in this work was a piece of a paper clip (Figure 1H). Using spring calipers the diameter of the wire was determined as .036 inches. Under the conditions of the example listed above the photographed paper clip would be 1.368 inches in diameter. To convert the measurement to centimeters multiply by 2.54 and to change the centimeters to microns multiply by 10,000.

THINGS TO COME IN SPACE PROGRAM

Out-of-this-world predictions by the National Aeronautics and Space Administration will put a man on the mmoon about ten years from now, fire an astronaut into orbit next year, and launch an orbiting astronomical observatory by 1964. Judging by past performance since January, 1959, the target dates for these goals and others set by the Government's civilian space agency may be achieved as predicted.

Early in 1959 NASA officials told a Senate committee that eight satellites and two deep-space probes would be launched in 1959, and six satellites and four deep-space probes in 1960.

In 1959, 12 satellites were successfully launched under NASA's auspices, of which one was a deep-space probe.

As of today, five of the six promised satellites have been fired into space, including Pioneer V, another deep-space probe. What is even more important, the 1960 U. S. satellites still are transmitting scientific data.

Also promised for this year are close to a hundred rocket soundings to an altitude of about 4,000 miles, and a sub-orbital launch of an astronaut in the nose-cone of a rocket.

There is some inclination to doubt the astronaut firing because NASA's program in this area called for several successful launch and recovery of animal-occupied rockets. There have not been enough of these to warrant rocketing a man into an earth orbit, it is believed.

NEW METHOD FOUND FOR LARGE RADIO TELESCOPES

A new method for "making" a large radio telescope has been developed by two astronomers at the Cavendish Laboratory in Cambridge, England. They say it can be made by mathematically combining the radio information received on two smaller radio telescopes.

The synthetic radio telescope was devised by Drs. M. Ryle and A. Hewish to obtain increased resolving power. Many investigations of the sources of radio waves in the heavens are limited by the resolving power that can be achieved by conventional methods of constructing the receiving antennas.

To overcome such limitations, Drs. Ryle and Hewish developed a method by which two antennas are so arranged that their relative positions can be altered to occupy successively the whole area of a much larger equivalent aerial. The results of such observations are then combined mathematically.

Investigations in Series and Parallel Circuit Combinations*

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In the December, 1957, issue of SCHOOL SCIENCE AND MATHEMATICS, there was an article by Harald C. Jensen describing a circuit that can be used to show series and parallel circuit combinations. My science teacher built the circuit and presented the following puzzle to the class: How many possible parallel and series combinations can be represented by the circuit? I enjoyed finding out not only how

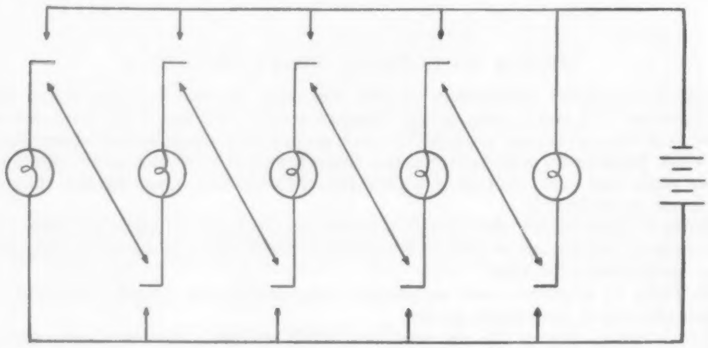

FIG. 1. The described circuit

many combinations there were for the original circuit which had four lights, but also investigating how many combinations there were for five, six, and more lights. This study eventually led to a formula which I believe will tell how many combinations there are for any number of lights. The formula turns out to be, if $C(n)$ represents the number of combinations for n number of lights,

$$C(n)=\frac{\left(\frac{1+\sqrt{5}}{2}\right)^{2n+1}-\left(\frac{1-\sqrt{5}}{2}\right)^{2n+1}}{\sqrt{5}}-1.$$

In the following article, I will describe how this formula was derived.

In my first analysis, I made use of a "tree" to represent the possible combinations. To construct the "tree" (Figure 2), first designate each light in the circuit by a letter. The first light is thus represented by a, the second by b, the third by c, and so on. Two or more lights in series are represented by putting the letters representing them next

* This is an article written by a high-school sophomore, based on his research for a Science Fair project. The editors are always pleased to publish student materials.

to each other. Thus lights *a* and *b* in series would be represented by *ab*. Lights *a*, *b*, and *c* in series would be represented by *abc*. In this discussion, a single light on will be considered a series combination containing one light, just as *abc* will be considered a series combination. Two series combinations in parallel will be represented by *abc-de* or *a-cd-e*.

It can be seen by studying the circuit diagram (Figure 1) that only adjacent lights in the circuit can be in series and that parallel combinations cannot be in series with other parallel combinations. Thus, these possibilities should not be counted.

Now let's consider series combinations only. They are, if a circuit of five lights is considered,

TABLE 1

a	*ab*	*abc*	*abcd*	*abcde*
b	*bc*	*bcd*	*bcde*	
c	*cd*	*cde*		
d	*de*			
e				

Each one of these represents a distinct combination of lights in series for the circuit. Series combinations can also be in parallel with light *a*. These combinations are

TABLE 2

	—b	*—bc*	*—bcd*	*—bcde*
Light *a* in parallel with	*—c*	*—cd*	*—cde*	
	—d	*—de*		
	—e			

Series combinations can also be in parallel with any series combination ending in *a* or *b*. These combinations are

TABLE 3

	—c	*—cd*	*—cde*
Any combination ending in *a*, or *b* in parallel with	*—d*	*—de*	
	—e		

The series combinations that can be in parallel with light *a*, *b*, or *c* are *d*, *de*, and *e*. The only light that can be in parallel with series combinations that end in *d* or a previous light is *e*. The preceding possibilities are summarized in the "tree diagram" that is shown in Figure 2.

It will be noticed that the series combinations comprising Table 1 are represented in column I of the tree. Each of these combinations

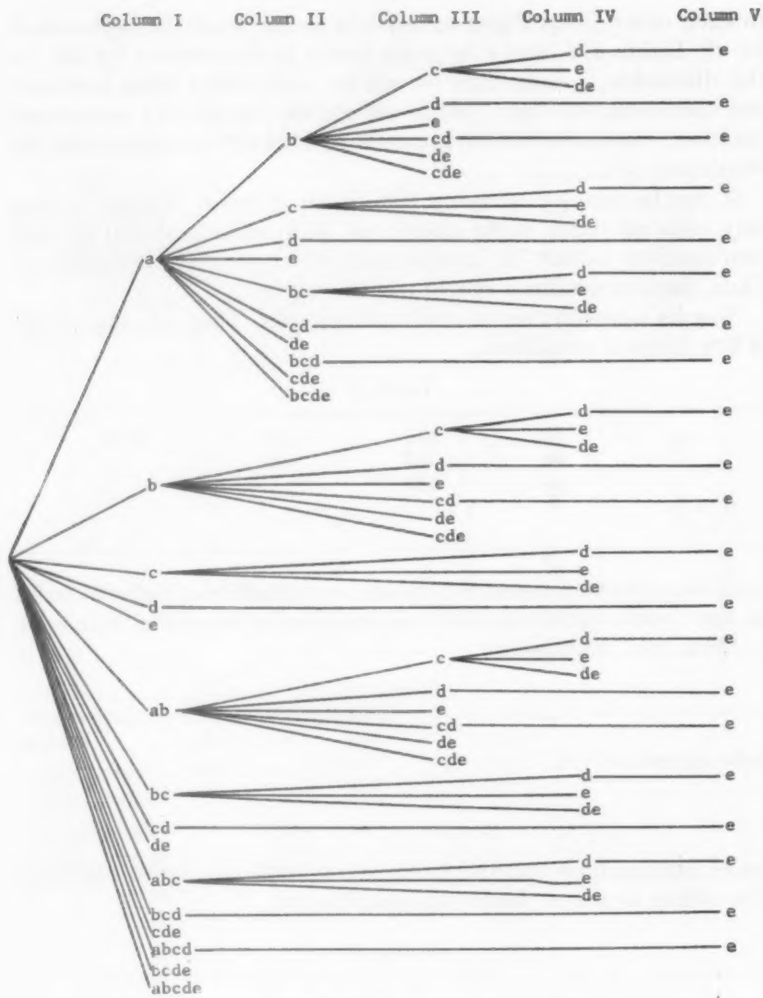

FIG. 2. Tree diagram for five lights.

represents a distinct circuit possibility for the circuit. The series combinations that comprise Table 2 are represented in column II. The group of combinations in column II is the possible combinations that are series combinations in parallel with light *a*. The group of series combinations in Table 3 are represented in column III. The top group represents the series combinations that are in parallel with the parallel combinations *a-b*. Thus the combination *de*, in the top group

of column III represents the distinct circuit possibility *a-b-de*, while the third *e* from the top of column V represents the distinct possible combination *a-b-cd-e*, and the fifth *e* from the top of column V represents the combination of *a-d-e*.

Now with the aid of Table 1, we see that there are $(1+2+3+4+5)$ possibilities in column I of Figure 2. There also are $(1+2+3+4)$ possibilities represented in column II. In column III, there are three groups of combinations, however, so the total number of combinations in this column is $3(1+2+3)$ or 18. Similarly, in column IV the group of combinations is represented eight times. Since there are $(1+2)$ combinations in each group, there is a total of $8(1+2)$ combinations in column IV. There are $21(1)$ or 21 combinations represented in column V. If we write these out, we have the total number of combination for five lights.

$$C(5)=1(1+2+3+4+5)+1(1+2+3+4)+3(1+2+3)+8(1+2)+21(1)$$

Consider the coefficients of the parenthetical expressions in the above formula. They are 1, 1, 3, 8, and 21. Upon examining these numbers, it is apparent that each number is the sum of all the preceding numbers of the series plus the preceding number again. Thus $1+1+3+3=8$ and $1+1+3+8+8=21$. A more important fact about the numbers is that they are, excepting the first "one," equal to every other term of the Fibonacci sequence.

The Fibonacci sequence is formed by taking the number one and following this rule: Each term of the sequence is the sum of the two preceding terms. The sequence is 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, $\cdots$. Since the Fibonacci sequence is necessary for the following discussion, I shall develop a few formulae about the sequence.

Any term of the Fibonacci sequence, F_n, is equal to the sum of the two preceding terms,

$$F_n=F_{n-1}+F_{n-2}.$$

But

$$F_{n-2}=F_{n-3}+F_{n-4}$$

and

$$F_{n-3}=F_{n-4}+F_{n-5}.$$

This process can be continued until

$$F_n=F_{n-1}+F_{n-3}+F_{n-5}+F_{n-7}\cdots F_3+F_1+F_0$$

if n is an even integer. Since F_0 is equal to zero, the rule can be stated: Any even numbered term of the Fibonacci sequence is equal

to the sum of all the odd numbered terms preceding it. Also by a similar process, if n is an odd integer,

$$F_n=F_{n-1}+F_{n-3}+F_{n-5}\cdots F_4+F_2+F_1.$$

For any series of elements in a specified order, if they are taken first one, then two, then three, at a time and so on up to n, and the order in which they are taken is the same as in the original series, there will be one combination ending in the first element of the series, two ending in the second element of the series, etc., until there are n combinations ending in the nth element of the series, so that there are $1(1+2+3+4\cdots n)$ possible ways of taking them.

As previously stated, there are $1(1+2+3+4+5)$ or 15 possible combinations represented in column I. In column II, there are $1(1+2+3+4)$ or 10 combinations represented. The reason the coefficient of the parenthetical expression is one is that there is only one combination in the previous series ending in the first term of the series, which is a. There are two numbers that end in b in the first column and one that ends in b in the second column. The result is that there are three sets of $(1+2+3)$ combinations in the third column.

To express these results in a form that will lead to generalization, we note that there are three combinations in column I that end in C, two in column II that end in c, and three in column III. These may be expressed as follows:

$$\begin{array}{l} 1+1+3 \\ 1+1 \\ 1. \end{array}$$

By substituting terms of the Fibonacci sequence for the numbers

$$\begin{array}{lll} F_1+F_2+F_4 & = & F_5 \\ F_1+F_2 & = & F_3 \\ F_1 & = & F_1 \\ \hline & & F_6. \end{array}$$

We have added horizontally, and then added vertically to get the sum of all the terms of the sequence represented in the array. The sum of $3F_1+2F_2+F_4=F_6$. This means there will be F_6 or 8 times the set $(1+2)$. By using the same method, we can show that $4F_1+3F_2+2F_4+F_6=F_7$, which is the number of combinations in column IV. By continuing this method, we can show that the coefficients of the parenthetical expressions given before will always be the even numbered terms of the Fibonacci sequence.

It is now possible to write a formula for the possible combinations for any number of lights.

$$C(n) = F_1(1+2+3 \cdots n) + F_2(1+2+3 \cdots n-1) + \cdots + F_{2n-4}(1+2) + F_{2n-2}(1). \tag{3}$$

The number of combinations for $n-1$ lights is then

$$C(n-1) = F_1(1+2+3 \cdots n-1) + F_2(1+2+3 \cdots n-2) + \cdots + F_{2n-6}(1+2) + F_{2n-4}(1). \tag{4}$$

Subtracting equation (4) from equation (3), we get

$$C(n) - C(n-1) = F_1(n) + F_2(n-1) + F_4(n-2) + \cdots + F_{2n-4}(2) + F_{2n-2}(1). \tag{5}$$

By arranging the terms on the right hand side of the equation in an array and adding as before, we get

$$C(n) - C(n-1) = F_{2n}. \tag{6}$$

Then

$$C(n) - C(n-2) = F_{2n} + F_{2n-2}. \tag{7}$$

By continuing this process, we get

$$C(n) - C(1) = F_{2n} + F_{2n-2} + F_{2n-4} + \cdots + F_6 + F_4. \tag{8}$$

Since n is an integer, $2n$ is an even integer so that

$$F_{2n} + F_{2n-2} + F_{2n-4} + \cdots + F_4 + F_2 + F_1 = F_{2n+1}. \tag{9}$$

By substituting equation (9) in equation (8), we get

$$C(n) - C(1) = F_{2n+1} - F_2 - F_1. \tag{10}$$

F_1, F_2 and $C(1)$ all equal one, so by substitution we get

$$C(n) = F_{2n+1} - 1. \tag{11}$$

It should be noticed that this does not include the possibility of all lights being off. If this possible combination is included, the formula becomes $C(n) = F_{2n+1}$. We will accept equation (11) as the equation that suits our purpose.

The formula for the nth term of the Fibonacci sequence is

$$F_n = \frac{\left(\frac{1+\sqrt{5}}{2}\right)^n - \left(\frac{1-\sqrt{5}}{2}\right)^n}{\sqrt{5}}. \tag{12}$$

Since we want the $2n+1$ term of the sequence, we substitute $2n+1$ for n in equation (12), and then substitute for F_{2n+1} in equation (11).

$$C(n)=\frac{\left(\frac{1+\sqrt{5}}{2}\right)^{2n+1}-\left(\frac{1-\sqrt{5}}{2}\right)^{2n+1}}{\sqrt{5}}-1. \tag{13}$$

This is the formula for the number of possible circuit combinations for n lights.

PROPOSED CHANGES IN CASMT BY-LAWS

The following changes in the By-Laws were proposed by the Board of Directors at the Spring Board Meeting in May. Action will be taken on these at the annual meeting in November.

ARTICLE III. OFFICERS
SECTION IV. POWER AND DUTIES OF OFFICERS:
(b) VICE-PRESIDENT:

Present	*Proposed*
He shall act for the President in the latter's absence. He shall also serve as a member of the Executive Committee.	He shall act for the President in the latter's absence. He shall serve as a member of the Executive Committee and shall act as Chairman of the Program Committee.

ARTICLE IV. BOARD OF DIRECTORS
SECTION IV. ELECTION, TENURE OF OFFICE, AND COMPENSATION:

Present	*Proposed*
. . . Those candidates declared elected to membership of the Board of Directors shall be the required number of nominees receiving the largest number of votes cast in the annual election as described in Section III (b) of Article III. . . .	The four nominees receiving the largest number of votes cast in the annual election as described in Section III (b) of Article III shall be declared elected to the Board of Directors. In case of a tie the decision shall be made by lot under the supervision of the Chairman of the Nominating Committee and the Secretary of the Association. . . .
. . . Vacancies in the Board of Directors or list of officers shall be filled by the Board of Directors at any meeting thereof. A director so chosen shall serve until the next annual business meeting when a successor shall be elected to fill the unexpired term.	. . . Vacancies in the Board of Directors or list of officers shall be filled by the Board of Directors. A director so chosen shall serve for the remainder of the unexpired term, when nomination and election shall take place as provided above.
Whenever directors are elected, whether at the expiration of a term or to fill vacancies, a certificate under the seal of the Association giving the names of those elected and the term of their office shall be recorded by the Treasurer and Business Manager in the office of the recorder of deeds where the certificate of organization is recorded.	The entire paragraph is deleted.

Highlights of the 1960 CASMT Convention

The 60th Convention of the Central Association of Science and Mathematics Teachers will be held at the Statler Hilton Hotel in Detroit, Michigan on November 24–26, 1960. The theme of the meeting is, "Challenging Science and Mathematics Students Toward the Horizons of Knowledge."

In keeping with this theme, the opening session on Thursday evening, November 24, considers, "The Challenge of Challenging Students." Initial discussion by a panel will be extended to include those in attendance.

This initial challenge will be extended, in the subsequent two days, to many other challenges and horizons of knowledge.

Horizons of Knowledge:

Our understanding of how we learn to interpret abstractions, the discovery of organized systems in the tiniest of particles, and pushing our explorations ever outward into the vast systems of outer space are stimulating examples of the ever expanding frontiers of knowledge. These "horizons" will be explored, in our general sessions with the following outstanding speakers:

"Horizons of Knowledge of the Atom"—Friday morning, November 25.

Dr. Leonard Roellig, Department of Physics, Wayne State University, Detroit, Michigan.

"Horizons of Knowledge of the Psychology of Mathematics—Elementary and Secondary"—Friday morning, November 25.

Dr. Henry Van Engen, Department of Mathematics, University of Wisconsin, Madison, Wisconsin.

"Horizons of Knowledge of Space"—Friday evening, November 25

Dr. F. D. Drake, National Radio Astronomy Observatory, Green Bank, West Virginia. Director, Project "Ozma."

"Horizons of Knowledge of the Cell"—Saturday morning, November 26.

Dr. Wayne E. Magee, Research Division, The Upjohn Company, Kalamazoo, Michigan.

In addition, a "horizon" of a different nature will be explained at the annual luncheon on Saturday, November 26. This is the horizon of a device—the teaching machine. "Teaching Machines on the Horizon? What Are the Implications for Science and Mathematics Teaching?" Dr. James Holland, Psychological Laboratory, Harvard University, Cambridge, Massachusetts.

PROBLEM DEPARTMENT

Conducted by Margaret F. Willerding

San Diego State College, San Diego, Calif.

This department aims to provide problems of varying degrees of difficulty which will interest anyone engaged in the study of mathematics.

All readers are invited to propose problems and to solve problems here proposed. Drawings to illustrate the problems should be well done in India ink. Problems and solutions will be credited to their authors. Each solution or proposed problem sent the Editor should have the author's name introducing the problem or solution *as on the following pages.*

The Editor of the Department desires to serve her readers by making it interesting and helpful to them. Address suggestions and problems to Margaret F. Wilderding, San Diego State College, San Diego, Calif.

SOLUTIONS AND PROBLEMS

Note. Persons sending in solutions and submitting problems for solutions should observe the following instructions.

1. Solutions should be in typed form, double spaced.

2. Drawings in India ink should be on a separate page from the solution.

3. Give the solution to the problem which you propose if you have one and also the source and any known references to it.

4. In general when several solutions are correct, the one submitted in the best form will be used.

LATE SOLUTIONS

2715, 2722. *Enoch J. Haga, Vacaville, Calif.*

2720. *Brother Felix John, Philadelphia, Pa.*

2722. *Mother Alphonsa Kohne, Arcadia, Mo.*

2700. *Proposed by J. B. Flansburg, Houston, Texas.*

Prove that there is no n for which the value of $\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+ \cdots +1/n$ is integral.

Comment and corrected solution by C. W. Trigg, Los Angeles City College.

The solution published on page 245 of the March 1960 issue of SCHOOL SCIENCE AND MATHEMATICS shows that the harmonic series is divergent, but it does not prove the proposed problem. A proof follows:

If the highest power of 2 occurring in any denominator of the series is 2^m, then that denominator is 2^m, for $2^m<2^{m+1}<2^mk$ for $k>2$. Hence the least common denominator of the series is 2^mx, where x is an odd number. When each term of the series is converted into a fraction having this L.C.D. as a denominator, every numerator will be even except that of $x/2^mx$. Therefore the sum of the numerators will be odd. Thus the sum, S_n, of the series is a fraction with an odd numerator and an even denominator hence cannot reduce to an integer.

This problem has previously been dealt with in the *American Mathematical Monthly*, Vol. 41, page 48, January 1934 where four references to solutions in foreign publications are given also.

A correction was also submitted by H. W. Gould, Morgantown, West Va.

2725. *Proposed by Louis J. Hall, Bloomington, Ind.*

There is an old problem in which a person beginning employment is given a choice of a pay schedule of starting play plus a raise of either \$150 a year or \$50 every six months, the raises to take effect at the end of each respective period of employment. It can be verified that the \$50 every six months is the better choice.

This is also true if the choices are \$200 and \$50 respectively, but not true if the choices are \$250 and \$50. Find an expression for a choice of "B" dollars raise per year of "A" dollars every six months which will be equivalent in total earnings in n years for n greater than number one.

Solution by Alan Wayne, Baldwin, New York.

Suppose that a constant raise of R_k dollars above the starting salary is given at the end of each $1/k$th part of a year, for $k=1, 2, 3$, and so on. Let E_k be the sum of all the moneys earned in n years from only these raises above the starting salary. Then

$$E_k = R_k + 2R_k + \cdots + (kn - 1)R_k,$$

so that

(1) $$E_k = \tfrac{1}{2}kn(kn - 1)R_k.$$

In particular, for yearly raises,

(2) $$E_1 = \tfrac{1}{2}n(n - 1)R_1.$$

When $n>1$, by using (1) and (2) we find that $E_1=E_k$ if and only if

(3) $$R_1 = [k^2 + k(k - 1)/(n - 1)]R_k.$$

For the problem, $k=2$, $R_1=B$, and $R_2=A$. Then

(4) $$B = [4 + 2/(n - 1)]A.$$

This is the desired relation. We note also that if $R_1<k^2 R_k$ for $n>1$, then $E_1<E_k$.

Solutions were also offered by Enoch J. Haga, Vacaville, Calif.; and Louis J. Hall, Bloomington, Ind.

2726. *Proposed by Lowell Van Tassel, San Diego, Calif.*

A neurologist wishes to calculate the total number of different ways that nerve cells can be linked pair wise, that is, two-by-two. For a small animal brain of only a million cells, he calculates.

$$10^{2,783,000}.$$

(a) Comment of his answer; (b) find a general solution for a brain of n (n even and very large) cells.

Solution by Alan Wayne, Baldwin, New York.

(a) In the given answer, neither the accuracy nor the precision of the result is stated. What is given is merely an order of magnitude.
(b) We shall solve the problem by two different methods.

Method I

Let $f(n-2)$ be the number of different sets of $(n-2)$ cells, for $n=4, 6, 8, \ldots$, in which each set consists of $\frac{1}{2}(n-2)$ pairs of linked cells, each pair disjoint from every other pair. Consider just one of these sets. Add 2 new cells to the set, making $\frac{1}{2}n$ possible pairs. To form new linked pairs, we have either to break up one of the existing pairs, or else to just link together the two newly added cells. To do anything else is to overlap in our subsequent enumeration of patterns. Hence the number of different patterns of $\frac{1}{2}n$ pairs formed from each set of $\frac{1}{2}(n-2)$ pairs is $2[\frac{1}{2}(n-2)]+1$; that is, $(n-1)$. Consequently, $f(n)=(n-1)f(n-2)$, and $f(2)=1$. These recursion relations lead to the relation

(1) $$f(n)=(n-1)(n-3)(n-5)\cdots(5)(3)(1), \qquad \text{for } n=2, 4, 6, \cdots.$$

This may be transformed to the form

(2) $$f(n) = n!/.2^{n/2}(n/2)!.$$

For n very large, Stirling's Approximation is $n! \approx \sqrt{2\pi n}\, n^n e^{-n}$, and this yields

$$f(n) \approx \sqrt{2}(n/e)^{n/2}. \tag{3}$$

For one million cells, $n=10^6$, and a calculation to five significant figures yields $f(10^6) \approx 1.4142 \times 10^{2,782,853}$.

Method II

We generalize the problem to determine the number of different patterns of disjoint m-chains possible from mk cells, for natural numbers $n=mk$. An m-chain is defined as a permutation of m cells in which two permutations are counted as different m-chains if an only if either permutation is not a reflection (reverse order) of the other. Then to form each pattern we must distribute the mk cells, m at a time, into k compartments all alike. If the compartments were all different, the number of patterns would be

$$\binom{mk}{m}\binom{mk-m}{m}\binom{mk-2m}{m}\cdots\binom{2m}{m}\binom{m}{m}(\tfrac{1}{2}m!),$$

where the last factor is used to count reflected permutations as the same. But since the compartments are all alike, we must divide the above expression by $k!$, and the result may be put into the form

$$f(n) = f(mk) = (mk)!/2k!(m!)^{k-1}. \tag{4}$$

Setting $m=2$, and $k=n/2$, we obtain the relation (2) above, and the rest is as before.

A solution was also offered by the proposer.

2727. *Proposed by W. R. Talbot, Jefferson City, Mo.*

Without the use of decimals determine whether the quantity $13\sqrt{5}-9\sqrt{10}+7\sqrt{17}-5\sqrt{34}$ is positive or negative.

Solution by C. W. Trigg, Los Angeles City College

Assume that

$$13\sqrt{5}-9\sqrt{10}+7\sqrt{17}-5\sqrt{34}>0,$$

then

$$\sqrt{5}(13-9\sqrt{2})>\sqrt{17}(5\sqrt{2}-7).$$
$$5(169-234\sqrt{2}+162)>17(50-70\sqrt{2}+49)$$
$$1655-1170\sqrt{2}>1683-1190\sqrt{2}$$
$$20\sqrt{2}>28$$
$$800>784$$

Therefore the assumption was correct, so the given quantity is positive.

Solutions were also offered by Merrill Barnebey, Tougaloo, Miss.; Robert Beckett, Calexico, Calif.; Felix John, Philadelphia, Pa.; Herbert R. Leifer, Pittsburgh, Pa.; Horace Mouser, Belleville, Mich.; Bernard T. Pleimann, Gardena, Calif.; Edith Robinson, Madison, Wis.; Alan Wayne, Baldwin, N. Y.; Dale Woods, Kirksville, M.; and the proposer.

2728. *Proposed by Cecil B. Read, University of Wichita, Wichita, Kans.*

Find a two digit number equal to the square of the ten's digit plus the square of the sum of its digits.

Solution by the proposer

We have to solve $t^2+(t+u)^2=10t+u$. Considering this as a quadratic in t, the roots are real if $25-u(u+8)$ is positive, which is true for $u=0, 1, 2$; the roots

are rational for $u=0$ or 1, but irrational if $u=2$. The possible numbers are 50 and 41.

Solutions were also submitted by Robert B. Beckett, Calexico, Calif.; Samuel D. Edwards, Andover, Mass.; Enoch J. Haga, Vacaville, Calif.; Felix John, Philadelphia, Pa.; Margaret Joseph, Milwaukee, Wis.; Mother Alphonsa Kohne, O.S.U., Arcadia, Mo.; Herbert R. Leifer, Pittsburgh, Pa.; Mart E. Mitchell, Plainfield, Ill.; Horace L. Mourer, Belleville, Mich.; Bernard T. Pleimann, Gardena, Calif.; W. R. Talbot, Jefferson City, Mo.; C. W. Trigg, Los Angeles, Calif.; Alan Wayne, Baldwin, N. Y.; Dale Woods, Kirksville, Mo.

2729. *Proposed by G. P. Speck, Virginia, Minnesota*

Prove or disprove the following conjecture: Given any real number r, for every $e>0$ there exists a pair of integers (m,n) such that $m-nr>e$.

Solution by Edith Robinson, Madison, Wis.

(1) The pair (0,0) is always a solution.

(2) If it is desired to find integers m and n with $m\neq 0$ and $n\neq 0$, these integers must fit the compound condition $m-nr<e$ *and* $m-nr<-e$. Now, since $-e<0<e$, integral solutions to $m-nr=0$ will do. This is equivalent to $m/n=r$, so if r is a rational number, there are infinitely many pairs of integers which will make $m-nr<e$.

However, if r is irrational, m and n cannot both be integers and $m/n=r$. But the problem does not require that $m-nr=0$, only that it be as close as we wish to 0, so that we need only find integers m and n such that m/n shall be as close as we wish to r. By the properties of the real numbers, such integers can be found.

Hence given the real number r, for every $e>0$, there exists a pair of integers (m,n) such that $m-nr<e$.

(3) The above discussion referred to the case where $r>0$; the argument is the same for $r<0$.

A solution was also offered by Alan Wayne, Baldwin, N. Y.

2730. *Proposed by Felix John, Philadelphia, Pa.*

Show that every prime factor is contained in $(n+r)!$ as often at least as it is contained in $n'\cdot !$

Solution by C. W. Trigg, Los Angeles City College.

It is well-known that the binomial coefficient $(n+r)!/n!r!$ is an integer. Hence the proposition.

This is the same as problem 2686 which appeared in the January 1960 issue.

A solution was also submitted by Bernard T. Pleimann, Gardena, Calif.; Alan Wayne, Baldwin, N. Y., and the proposer.

STUDENT SOLUTIONS

S-3. Proposed by Lowell Van Tassel, San Diego, Calif.

Calculate, in the duodecimal system (radix$=12$, $10=t$, $11=e$) the following values, retaining results in the duodecimal system. (Remark: It is considered cheating to multiply the translated problems in the decimal system and then re-convert.)

(a) $(\text{tete})^2$

(b) $(\text{ete})^3$

(c) $(\text{tete})\times(\text{ete})$

(d) $(\text{toe})^6$

(e) $\dfrac{\text{toot}}{\text{tet}}$

Solution by Samuel D. Edwards, Andover, Mass.

(a) $(\text{tete})^2 =$

$$\begin{array}{r}
\text{tete}\\
\times\ \text{tete}\\
\hline
\text{t0e01}\\
\text{91e12}\\
\text{t0e01}\\
\text{91e12}\\
\hline
\text{t0t02121}
\end{array}\quad \textit{Ans.}$$

(b) $(\text{ete})^3 =$

$$\begin{array}{r}
\text{ete}\\
\times\ \text{ete}\\
\hline
\text{te01}\\
\text{9e12}\\
\text{te01}\\
\hline
\text{e9t121}
\end{array}
\qquad\qquad
\begin{array}{r}
\text{e9t121}\\
\times\ \text{ete}\\
\hline
\text{tt030te}\\
\text{9t24e8t}\\
\text{tt030te}\\
\hline
\text{e8936188e}
\end{array}\quad \textit{Ans.}$$

(c) $\text{tete} \times \text{ete} =$

$$\begin{array}{r}
\text{tete}\\
\times\ \text{ete}\\
\hline
\text{t0e01}\\
\text{91e12}\\
\text{t0e01}\\
\hline
\text{tte0121}
\end{array}\quad \textit{Ans.}$$

(d) $(\text{t0e})^5$

$$\begin{array}{r}
\text{toe}\\
\times\ \text{toe}\\
\hline
\text{92t1}\\
\text{84920}\\
\hline
\text{8564t1}
\end{array}
\qquad
\begin{array}{r}
\text{8564t1}\\
\times\ \text{8564t1}\\
\hline
\text{8564t1}\\
\text{707404t}\\
\text{29t1744}\\
\text{4292506}\\
\text{3638025}\\
\text{5783288}\\
\hline
\text{5e710t8t5181}
\end{array}
\qquad
\begin{array}{r}
\text{5e710t8t5181}\\
\times\ \text{t0e}\\
\hline
\text{5575e9t16864e}\\
\text{4e7tt8e48348t0}\\
\hline
\text{501462e264e544e}
\end{array}\quad \textit{Ans.}$$

(e)

$$\frac{\text{toot}}{\text{tet}} = \text{tet}\,\overline{)\,\text{toot}}\ \text{(quotient t)};\quad \begin{array}{r}\text{toot}\\ \text{91t4}\\ \hline \text{t26}\end{array}\qquad = \text{t}\,\frac{\text{t26}}{\text{tet}}\quad \textit{Ans.}$$

S-4. Taken from Mathematical Pi.

The Geometry teacher, feeling the need for some fresh air, glanced at the clock as he went out at some time between 4 and 5 P.M.

On his return about 3 hours later, he noticed that the hands of the clock had exactly changed places. What was the time of his departure?

Solution by Enoch J. Haga, Vacaville, California.

Since the teacher left between 4 and 5 P.M., he must have returned between 7 and 8 P.M. As the little hand was limited to the face of the clock between 4 and 5 when the teacher departed, it was likewise limited to the face of the clock between 7 and 8 when the teacher returned. As the hands changed position, the teacher left the room at 4:37 P.M. and returned at 7:23 P.M. The teacher was out of the room for 2 hours and 46 minutes.

STUDENT HONOR ROLL

The Editor will be very happy to make special mention of classes, clubs, or individual students who offer solutions to problems submitted in this department. Teachers are urged to report to the Editor such solutions.

Editor's Note: For a time each student contributor will receive a copy of the magazine in which his name appears.

For this issue the Honor Roll appears below.

2727, 2728, 2729, 2730. *Lee. Mitchell, Glencoe, Ill.*

2728. *Floyd D. Wilder, Nazarene College, Bethany, Okla.*

PROBLEMS FOR SOLUTION

2749. *Proposed by G. P. Speck, Virginia, Minn.*

If a spoke of a wheel is initially perpendicular to the horizontal, determine the area cut by the spoke as the wheel rolls along the horizontal completing one revolution.

2750. *Proposed by Robert L. Guderjohn, Eugene, Ore.*

How many ways will m like balls go into n different boxes?

2751. *Proposed by C. W. Trigg, Los Angeles, Calif.*

The midpoints of the sides of an equilateral triangle are joined, thus forming an assemblage of nine equal line segments. (a) Starting at a particular vertex, how many continuous paths may be followed by traversing each segment once only while returning to the starting point? (b) How many essentially different path patterns are involved?

2752. *Proposed by Donald R. Byrkit, West Chicago, Ill.*

The thirteenth of the month is more apt to be Friday than any other day of the week. Prove that this is true or false.

2753. *Proposed by Walter H. Carnahan, Madison, Ind.*

I was riding on a train from Chicago to New York. I called the porter and asked for a table to be placed in my roomette so that I could work on a manuscript. When he set up the table I laid out some mathematics manuscript and began to work. He saw the numbers and sketches but made no remark. However later he returned, laid a piece of paper on my table, and said, "Suh, can you tell me how come that is so? A man he give me them numbahs." On the paper, was written the expression $342=97$. Now, I ask you to explain how come?

2754. *Proposed by Enoch J. Haga, Vacaville, Calif.*

An accounting student had the multiplication to perform:

$$\begin{array}{r} 219912000 \\ \underline{64} \end{array}.$$

Mystified to learn that the product exactly equaled the multiplicand, he rechecked his work and found that he had made no mistake in his multiplication. What was his error?

Depleted uranium can be used in the steel industry, in the construction of shielding materials and in various alloys it is believed. Stocks of depleted uranium, from which the fissionable isotope U-235 has been extracted, are accumulating at a rate of tens of millions of pounds a year.

Metallurgists state that uranium is still too expensive to compete with metals now used in the steel and alloy industries, but that the price might be lowered if increased demand were to lead to better methods of preparation.

Books and Teaching Aids Received

PHYSICS (Part I and II), by Robert Resnick, *Professor of Physics, Rensselaer Polytechnic Institute* and David Halliday, *Professor of Physics, University of Pittsburgh*. Both cloth. Both 23.5×15.5 cm. 1960. John Wiley and Sons, Inc., 440 Fourth Avenue, New York 16, N. Y.

PHYSICS, Part I. Pages 554+40.
PHYSICS, Part II. Pages 1025+40.

ANALYTIC TRIGONOMETRY, by Paul S. Mostert. Cloth. Pages x+165. 21×14.5 cm. 1960. Prentice-Hall, Inc., 70 Fifth Ave., New York 11, N. Y. Price $3.95.

ANALYSIS OF RESEARCH IN THE TEACHING OF MATHEMATICS, by Kenneth E. Brown, *Specialist for Mathematics, Office of Education* and John J. Kinsella, *School of Education, New York University*. Paper. 50 pages. 23×15 cm. Bulletin 1960, No. 8. U. S. Department of Health, Education, and Welfare, Washington, D. C.

RETHINKING SCIENCE EDUCATION, Prepared by the Yearbook Committee, Edited by Nelson Henry. Paper. 23×15 cm. Pages xviii 344. 1960. The University of Chicago Press, Chicago, Illinois.

FROM ZERO TO INFINITY, by Constance Reid. Cloth. 20.5×13.5 cm. 161 pages. 1960. Thomas Y. Crowell Company, 432 Fourth Avenue, New York, N. Y. Price $3.95.

STRUCTURE AND CHANGE, by G. S. Christiansen and Paul H. Garrett. Cloth. 608 pages. 25.5×18.5 cm. 1960. W. H. Freeman and Company, 660 Market St., San Francisco 4, California. Price $8.75.

FIGURES OF EQUILIBRIUM OF CELESTIAL BODIES, by Zdenek Kopal. Cloth. 135 pages. 23.5×15 cm. 1960. The University of Wisconsin Press, 811 State Street, Madison, Wisconsin. Price $3.00.

NAIVE SET THEORY, by Paul R. Halmos, *Professor of Mathematics, The University of Chicago*. Cloth. Pages vii 104. 23.5×15.5 cm. 1960. D. Van Nostrand Company, Inc., 120 Alexander Street, Princeton, New Jersey. Price $3.50.

THE MAGIC OF RUBBER, by E. Joseph Dreany. Cloth. 92 pages. 21.5×16 cm. 1960. G. P. Putnam's Sons, 210 Madison Avenue, New York, N. Y. Price $2.75.

A BIBLIOGRAPHY OF REFERENCE BOOKS FOR ELEMENTARY SCIENCE, by George G. Mallinson, *Western Michigan University, Kalamazoo, Michigan*, and Jacqueline V. Buck, Formerly, *Grosse Pointe Public Schools, Grosse Pointe, Michigan*. Paper. 28×21.5 cm. 40 pages. 1960. The National Science Teachers Association, 1201 Sixteenth Street, NW, Washington, D. C. Price 50 cents.

HEREDITY AND HUMAN NATURE, by David C. Rife. Cloth. 265 pages. 20.5×13 cm. 1960. Vantage Press, Inc., 120 W. 31 Street, New York 1, N. Y. Price $4.50.

FREE AND INEXPENSIVE LEARNING MATERIALS. Paper. 252 pages. 21.5×14 cm. 1960. Division of Surveys and Field Services, George Peabody College for Teachers, Nashville 5, Tennessee. Price $1.50.

MODERN FOREIGN LANGUAGES IN HIGH SCHOOL, by Patricia O'Connor, *Consultant for Foreign Languages*. Paper. 49 pages. 23.5×15 cm. Bulletin 1960, No. 9. U. S. Department of Health, Education, and Welfare, Office of Education, Washington, D.C.

I LIKE BUTTERFLIES, by Gladys Conklin. Cloth. 25 pages. 25×18 cm. 1960. Holiday House, 8 West 13th Street, New York 11, N. Y. Price $2.95.

CAN I BE A TECHNICIAN? Paper. 16 pages. 21×14 cm. 1960. General Motors Corporation Rublic Relations Staff, P.O. Box 177, Detroit 2, Michigan.

HANDICRAFT AND ART. Paper. 8 pages. 21×14 cm. 1960. John Murray, 50 Albemarle St., London W.I., England.

REPORT OF THE PRESIDENT. Paper. 53 pages. 23×15.5 cm. Academic Year 1958–1959. Carnegie Institute of Technology, Pittsburgh 13, Pennsylvania.

A DOVER SCIENCE SAMPLER, Edited by George Barkin. Paper. 64 pages. 13.5 ×20.5 cm. 1960. Dover Publications, Inc., 180 Varick Street, New York 14, N. Y.

ADMIRAL RICHARD E. BYRD, by Alfred Steinberg. Cloth. 20.5×14 cm. 128 pages. 1960. G. P. Putnam's Sons, 210 Madison Ave., New York, N. Y. Price $2.50.

FIRST PRINCIPLES OF ATOMIC PHYSICS, by Richard F. Humphreys, *Formerly Associate Professor of Physics* and Robert Beringer, *Assistant Professor of of Physics, Yale University*. Cloth. ix+390 pages. 15.5×23.5 cm. 1960. Harper and Brothers, 49 East 33rd St., New York 16, New York.

HARPER TORCHBOOKS SERIES. All paper. All 13.5×20 cm. 1960. Harper and Brothers, 49 East 33rd Street, New York 16, N. Y.
PHYSICS AND MICROPHYSICS, by Louis de Broglie. 286 pages. Price $1.50.
FROM ATOMS TO ATOM, by Andrew G. Van Melsen. 240 pages. Price $1.45.
MOLECULES IN MOTION, by T. G. Cowling. 183 pages. Price $1.35.
SCIENTIFIC EXPLANATION, by R. B. Braithwaite. 374 pages. Price $1.85.
THE PHILOSOPHY OF SCIENCE, by Stephen Toulmin. 176 pages. Price $1.25.
MATHEMATICS IN ACTION, by O. Sutton. 236 pages. Price $1.45.

BASIC CONCEPTS OF ELEMENTARY MATHEMATICS, by William L. Schaaf. Cloth. xvii 386 pages. 15.5×22 cm. 1960. John Wiley and Sons, Inc., 440 Fourth Avenue, New York 16, N. Y. Price $5.50.

ANN ARBOR SCIENCE PAPERBACKS SERIES. All paper. All 13.5×20 cm. 1960. The University of Michigan Press, Ann Arbor, Michigan. All $1.95.
THE ANTS, by Wilhelm Goetsch.
THE STARS, by W. Kruse and W. Dickevoss.
THE SENSES, by Wolfgang von Buddenbrock.
LIGHT: VISIBLE AND INVISIBLE, by Eduard Ruechardt.
THE BIRDS, by Oskar and Katharina Heinroth.
ERB AND FLOW: The Tides of Earth, Air and Water, by Albert Defant.
ANIMAL CAMOUFLAGE, by Adolf Portmann.
PLANET EARTH, by Karl Stumpff.
THE SUN, by Karl Kiepenheuer.
VIRUS, by Wolfhard Weidel.

THE OSCILLATING UNIVERSE, by Ernst J. Opik. Paper. 144 pages. 18.5×11 cm. 1960. New American Library of World Literature, Inc., 501 Madison Avenue, New York 22, New York. Price $.50.

GEOMETRY, by Charles F. Brumfiel, *Ball State Teachers College*, Robert E. Eicholz, *Ball State Teachers College*, and Merrill E. Shanks, *Purdue University*. Cloth. Pages xi, 288. 23×15 cm. 1960. Addison-Wesley Publishing Company, Inc., Reading, Massachusetts. Price $4.75.

FUNDAMENTALS OF MATHEMATICS, by E. P. Vance, *Oberlin College*. Cloth. Pages x 469. 15×23 cm. 1960. Addison-Wesley Publishing Company, Inc., Reading, Massachusetts. Price $7.50.

CALCULUS AND ANALYTIC GEOMETRY (3rd Edition), by George B. Thomas,

Massachusetts Institute of Technology. Pages xii 1010. 23×15 cm. 1960. Addison Wesley Publishing Company, Inc., Reading, Massachusetts. Price $9.75.

SCIENCE/A SEARCH FOR EVIDENCE, by J. Darrell Barnard, *New York University* Celia Stendler, *University of Illinois*, Benjamin Spock, M. D., *Western Reserve University , Cleveland*, and Nelson F. Beeler, *State University College of Education, Potsdam, New York*. Cloth. Pages viii 422. 23.5×18 cm. 1960. The Macmillan Company, 60 Fifth Avenue, New York 11, New York. Price $3.84.

SCIENCE/A WAY TO SOLVE PROBLEMS, by J. Darrel Barnard, *New York University*, Celia Stendler, *University of Illinois*, Benjamin Spock, M.D., *Western Reserve University, Cleveland*, and Nelson F. Beeler, *State University College of Education, Potsdam, New York*. Cloth. Pages ix 422. 23.5×18 cm. 1960. The Macmillan Company, 60 Fifth Avenue, New York 11, New York. Price $3.96.

ELEMENTS OF GENERAL CHEMISTRY, by Jay A. Young. Cloth. Pages viii 466. 14.5×23 cm. 1960. Prentice-Hall, Inc., 70 Fifth Ave., New York 11, N. Y. Price $6.95.

INTERMEDIATE ALGEBRA, by Roy Dubisch, Vernon E. Howes, Steven J. Bryant. Cloth. Pages xii 286. 14.5×23 cm. 1960. John Wiley & Sons, Inc., 440 Fourth Ave., New York 16, New York. Price $4.50.

THE NATURE OF SCIENCE, by David Greenwood. Cloth. Pages xiii 94. 18.5×11 cm. 1960. Philosophical Library, Inc., 15 East 40 Street, New York 16, New York. Price $3.75.

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ALGEBRA AND TRIGONOMETRY, by Edward A. Cameron, *University of North Carolina*. Cloth. Pages xi 290. 23.5×15 cm. 1960. Henry Holt and Company, 383 Madison Avenue, New York 17, N. Y. Price $5.00.

Book Reviews

PHYSICS, FOUNDATIONS AND FRONTIERS by George Gamow and John M. Cleveland. Cloth. 1960. v–xviii+551 pages. Prentice-Hall, Inc., Englewood Cliffs, New Jersey. 15×23 cm.

Physics, Foundations and Frontiers is an introductory-college-physics textbook. It is written primarily for students in liberal arts programs.

The authors have attempted to present topics traditionally covered in college as well as present a considerable amount of new approach in a course of this kind. Much of the material has been taken from Dr. Gamow's physical science book *Matter, Earth, and Sky*. However, most of the material is new.

The mathematics has been kept as such a level that a student with a good background in high-school algebra should follow the derivation of equations and formulas.

The textbook is well illustrated and the drawings are excellent. The test material is readable, interesting and written only as these authors could write it.

It is the opinion of this reviewer that this would be an excellent textbook for students in liberal arts. The student would have a considerable background in classical as well as modern physics.

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SATURDAY SCIENCE by Scientists of the Westinghouse Research Laboratories. 1960. Cloth. 16×24 cm. 333 pages. E. P. Dutton & Company, Inc., New York.

Saturday Science is the outgrowth of a series of lectures presented in the Westinghouse Science Honors Institute since 1957. The lectures were presented to approximately two hundred high-school students from the Pittsburgh area.

The topics included in the book are: Light, Electrons and Crystals; Atomic Collision and Gas Discharge; Atomic Nuclei and Radioactivity; Low Temperature Phenomena; Chemistry of Solids; Physics of Metals; Microstructure and Heat Treatment of Metals; Surface Filmson Metals; Propulsion; Mathematics, Experimental Science; Inhuman Arithmetic: Maxima and Minima in Physical Processes; Modern Analytical Methods; and Interdisciplinary Nature of Science. Obviously, the areas covered are not ordinarily treated to a great extent in either high school or college level.

The book contains excellent diagrams and is well illustrated and interestingly written.

This book would be an excellent addition to any science library. It might also be useful to the science teacher to supplement his lectures and bring him up to date on recent advances in the physical sciences.

J. Bryce Lockwood